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MATHEMATICAL TABLES

CHIEFLY TO FOUR FIGURES

FIRST SERIES

BY

JAMES MILLS PEIRCE

UNIVERSITY PROFESSOR OF MATHEMATICS IN HARVARD
UNIVERSITY

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Logarithms.

N											P. P.				
	0	1	2	3	4	5	6	7	8	9	1.	2.	3.	4.	5.
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4.	8.	12.	17.	21
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11.	15.	19
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3.	7.	10.	14.	17
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6.	10.	13.	16
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3.	6.	9.	12.	15
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3.	6.	8.	11.	14
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3.	5.	8.	11	13
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2.	5.	7.	10.	12
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2.	5.	7.	9.	12
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2.	4.	7.	9.	11
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2.	4.	6.	8	11
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2.	4.	6.	8	10
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2.	4.	6.	8	10
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2.	4.	5.	7.	9
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2.	4.	5.	7.	9
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2.	3.	5.	7.	9
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2.	3.	5.	7.	8
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2.	3.	5.	6.	8
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2.	3.	5.	6.	8
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1.	3.	4.	6.	7
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1.	3.	4.	6.	7
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1.	3.	4.	6.	7
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1.	3.	4.	5.	7
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1.	3.	4.	5.	6
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1.	3.	4.	5.	6
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1.	2.	4.	5.	6
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1.	2.	4.	5.	6
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1.	2.	3.	5.	6
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1.	2.	3.	5.	6
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1.	2.	3.	4.	6
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1.	2.	3.	4.	5
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1.	2.	3.	4.	5
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1.	2.	3.	4.	5
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1.	2.	3.	4.	5
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1.	2.	3.	4.	5
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1.	2.	3.	4.	5
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1.	2.	3.	4.	5
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1.	2.	3.	4.	5
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1.	2.	3.	4.	4
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1.	2.	3.	4.	4
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1.	2.	3.	3.	4
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1.	2.	3.	3.	4
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1.	2.	2.	3.	4
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1.	2.	2.	3.	4
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1.	2.	2.	3.	4

Logarithms

N											P. P.
	0	1	2	3	4	5	6	7	8	9	1. 2. 3. 4. 5
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1. 2. 2. 3. 4
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1. 2. 2. 3. 4
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1. 2. 2. 3. 4
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1. 1. 2. 3. 4
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1. 1. 2. 3. 4
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1. 1. 2. 3. 4
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1. 1. 2. 3. 4
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1. 1. 2. 3. 3
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1. 1. 2. 3. 3
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1. 1. 2. 3. 3
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1. 1. 2. 3. 3
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1. 1. 2. 3. 3
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1. 1. 2. 3. 3
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1. 1. 2. 3. 3
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1. 1. 2. 3. 3
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1. 1. 2. 2. 3
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1. 1. 2. 2. 3
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1. 1. 2. 2. 3
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1. 1. 2. 2. 3
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1. 1. 2. 2. 3
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1. 1. 2. 2. 3
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1. 1. 2. 2. 3
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1. 1. 2. 2. 3
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1. 1. 2. 2. 3
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1. 1. 2. 2. 3
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1. 1. 2. 2. 3
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1. 1. 2. 2. 3
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1. 1. 2. 2. 3
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1. 1. 2. 2. 3
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1. 1. 2. 2. 3
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1. 1. 2. 2. 3
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1. 1. 2. 2. 3
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0. 1. 1. 2. 2
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0. 1. 1. 2. 2
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0. 1. 1. 2. 2
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0. 1. 1. 2. 2
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0. 1. 1. 2. 2
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0. 1. 1. 2. 2
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0. 1. 1. 2. 2
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0. 1. 1. 2. 2
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0. 1. 1. 2. 2
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0. 1. 1. 2. 2
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0. 1. 1. 2. 2
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0. 1. 1. 2. 2
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0. 1. 1. 2. 2

Logarithms.

N	0	1	2	3	4	5	6	7	8	9	10
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	0043
101	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	0086
102	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	0128
103	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	0170
104	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	0212
105	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	0253
106	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	0294
107	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	0334
108	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	0374
109	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	0414
110	0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	0453
111	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	0492
112	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	0531
113	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	0569
114	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	0607
115	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	0645
116	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	0682
117	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	0719
118	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	0756
119	0756	0759	0763	0766	0770	0774	0777	0781	0785	0788	0792
120	0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	0828
121	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	0864
122	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	0899
123	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	0934
124	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	0969
125	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	1004
126	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	1038
127	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	1072
128	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103	1106
129	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	1139
130	1139	1143	1146	1149	1153	1156	1159	1163	1166	1169	1173
131	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202	1206
132	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235	1239
133	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268	1271
134	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300	1303
135	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332	1335
136	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364	1367
137	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396	1399
138	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427	1430
139	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458	1461
140	1461	1464	1467	1471	1474	1477	1480	1483	1486	1489	1492
141	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520	1523
142	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550	1553
143	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581	1584
144	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611	1614
145	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641	1644
146	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670	1673
147	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700	1703
148	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729	1732
149	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758	1761

Logarithms.

N	0	1	2	3	4	5	6	7	8	9	10
150	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	1790
151	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	1818
152	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	1847
153	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	1875
154	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	1903
155	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	1931
156	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	1959
157	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	1987
158	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	2014
159	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	2041
160	2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	2068
161	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	2095
162	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	2122
163	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	2148
164	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	2175
165	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	2201
166	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	2227
167	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	2253
168	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	2279
169	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	2304
170	2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	2330
171	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	2355
172	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	2380
173	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	2405
174	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2430
175	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	2455
176	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	2480
177	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	2504
178	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	2529
179	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	2553
180	2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	2577
181	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	2601
182	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	2625
183	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	2648
184	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	2672
185	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	2695
186	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	2718
187	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	2742
188	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	2765
189	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	2788
190	2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	2810
191	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	2833
192	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	2856
193	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	2878
194	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	2900
195	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	2923
196	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	2945
197	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	2967
198	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	2989
199	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	3010

Logarithms of Sums and Differences.

A	6.	7.	8.	9.	0.	1.	2.	3.
00	0.0000	0.0004	0.0043	0.0414 9	0.3010 50	1.0414 91	2.0043	3.0004
01	0.0000	0.0004	0.0044	0.0423 9	0.3061 51	1.0505 91	2.0142	3.0104
02	0.0000	0.0005	0.0045	0.0432 9	0.3111 51	1.0596 91	2.0241	3.0204
03	0.0000	0.0005	0.0046	0.0442 10	0.3163 52	1.0687 91	2.0340	3.0304
04	0.0000	0.0005	0.0047	0.0452 10	0.3215 52	1.0779 92	2.0439	3.0404
05	0.0000	0.0005	0.0048	0.0462 10	0.3267 53	1.0871 92	2.0539	3.0504
06	0.0000	0.0005	0.0050	0.0472 10	0.3321 53	1.0963 92	2.0638	3.0604
07	0.0001	0.0005	0.0051	0.0482 11	0.3374 54	1.1055 92	2.0737	3.0704
08	0.0001	0.0005	0.0052	0.0493 11	0.3429 55	1.1147 92	2.0836	3.0804
09	0.0001	0.0005	0.0053	0.0504 11	0.3484 55	1.1239 92	2.0935	3.0904
10	0.0001	0.0005	0.0054	0.0515 11	0.3539 56	1.1332 93	2.1034	3.1003
11	0.0001	0.0006	0.0056	0.0526 11	0.3595 56	1.1425 93	2.1134	3.1103
12	0.0001	0.0006	0.0057	0.0538 12	0.3652 57	1.1518 93	2.1233	3.1203
13	0.0001	0.0006	0.0058	0.0550 12	0.3709 57	1.1611 93	2.1332	3.1303
14	0.0001	0.0006	0.0060	0.0562 12	0.3766 58	1.1704 93	2.1431	3.1403
15	0.0001	0.0006	0.0061	0.0574 12	0.3825 59	1.1797 93	2.1531	3.1503
16	0.0001	0.0006	0.0062	0.0586 13	0.3884 59	1.1891 94	2.1630	3.1603
17	0.0001	0.0006	0.0064	0.0599 13	0.3943 60	1.1984 94	2.1729	3.1703
18	0.0001	0.0007	0.0065	0.0612 13	0.4003 60	1.2078 94	2.1829	3.1803
19	0.0001	0.0007	0.0067	0.0625 13	0.4063 61	1.2172 94	2.1928	3.1903
20	0.0001	0.0007	0.0068	0.0639 14	0.4124 61	1.2266 94	2.2027	3.2003
21	0.0001	0.0007	0.0070	0.0653 14	0.4186 62	1.2360 94	2.2127	3.2103
22	0.0001	0.0007	0.0071	0.0667 14	0.4248 62	1.2454 94	2.2226	3.2203
23	0.0001	0.0007	0.0073	0.0681 15	0.4311 63	1.2548 94	2.2325	3.2303
24	0.0001	0.0008	0.0075	0.0696 15	0.4374 63	1.2643 95	2.2425	3.2402
25	0.0001	0.0008	0.0077	0.0711 15	0.4438 64	1.2738 95	2.2524	3.2502
26	0.0001	0.0008	0.0078	0.0726 15	0.4502 65	1.2832 95	2.2624	3.2602
27	0.0001	0.0008	0.0080	0.0742 16	0.4567 65	1.2927 95	2.2723	3.2702
28	0.0001	0.0008	0.0082	0.0757 16	0.4632 66	1.3022 95	2.2823	3.2802
29	0.0001	0.0008	0.0084	0.0774 16	0.4698 66	1.3117 95	2.2922	3.2902
30	0.0001	0.0009	0.0086	0.0790 17	0.4764 67	1.3212 95	2.3022	3.3002
31	0.0001	0.0009	0.0088	0.0807 17	0.4831 67	1.3308 95	2.3121	3.3102
32	0.0001	0.0009	0.0090	0.0824 17	0.4899 68	1.3403 95	2.3221	3.3202
33	0.0001	0.0009	0.0092	0.0841 18	0.4966 68	1.3499 95	2.3320	3.3302
34	0.0001	0.0009	0.0094	0.0859 18	0.5035 69	1.3594 96	2.3420	3.3402
35	0.0001	0.0010	0.0096	0.0877 18	0.5104 69	1.3690 96	2.3519	3.3502
36	0.0001	0.0010	0.0098	0.0896 19	0.5173 70	1.3786 96	2.3619	3.3602
37	0.0001	0.0010	0.0101	0.0915 19	0.5243 70	1.3881 96	2.3718	3.3702
38	0.0001	0.0010	0.0103	0.0934 19	0.5313 71	1.3977 96	2.3818	3.3802
39	0.0001	0.0011	0.0105	0.0953 20	0.5384 71	1.4073 96	2.3918	3.3902
40	0.0001	0.0011	0.0108	0.0973 20	0.5455 72	1.4170 96	2.4017	3.4002
41	0.0001	0.0011	0.0110	0.0993 20	0.5527 72	1.4266 96	2.4117	3.4102
42	0.0001	0.0011	0.0113	0.1014 21	0.5599 72	1.4362 96	2.4216	3.4202
43	0.0001	0.0012	0.0115	0.1035 21	0.5672 73	1.4458 96	2.4316	3.4302
44	0.0001	0.0012	0.0118	0.1057 22	0.5745 73	1.4555 96	2.4416	3.4402
45	0.0001	0.0012	0.0121	0.1078 22	0.5819 74	1.4651 97	2.4515	3.4502
46	0.0001	0.0013	0.0123	0.1101 22	0.5893 74	1.4748 97	2.4615	3.4602
47	0.0001	0.0013	0.0126	0.1123 23	0.5967 75	1.4845 97	2.4715	3.4701
48	0.0001	0.0013	0.0129	0.1146 23	0.6042 75	1.4941 97	2.4814	3.4801
49	0.0001	0.0013	0.0132	0.1169 24	0.6118 76	1.5038 97	2.4914	3.4901
50	0.0001	0.0014	0.0135	0.1193 24	0.6193 76	1.5135 97	2.5014	3.5001

Logarithms of Sums and Differences.

A	6.	7.	8.	9.	0.	1.	2.	3.
50	0.0001	0.0014	0.0135	0.1193 24	0.6193 76	1.5135 97	2.5014	3.5001
51	0.0001	0.0014	0.0138	0.1218 24	0.6269 76	1.5232 97	2.5113	3.5101
52	0.0001	0.0014	0.0141	0.1242 25	0.6346 77	1.5329 97	2.5213	3.5201
53	0.0001	0.0015	0.0145	0.1267 25	0.6423 77	1.5426 97	2.5313	3.5301
54	0.0002	0.0015	0.0148	0.1293 26	0.6501 78	1.5523 97	2.5413	3.5401
55	0.0002	0.0015	0.0151	0.1319 26	0.6578 78	1.5621 97	2.5512	3.5501
56	0.0002	0.0016	0.0155	0.1345 27	0.6657 78	1.5718 97	2.5612	3.5601
57	0.0002	0.0016	0.0158	0.1372 27	0.6735 79	1.5815 97	2.5712	3.5701
58	0.0002	0.0016	0.0162	0.1399 28	0.6814 79	1.5913 97	2.5811	3.5801
59	0.0002	0.0017	0.0166	0.1427 28	0.6893 80	1.6010 97	2.5911	3.5901
60	0.0002	0.0017	0.0170	0.1455 28	0.6973 80	1.6108 98	2.6011	3.6001
61	0.0002	0.0018	0.0173	0.1484 29	0.7053 80	1.6205 98	2.6111	3.6101
62	0.0002	0.0018	0.0177	0.1513 29	0.7134 81	1.6303 98	2.6210	3.6201
63	0.0002	0.0018	0.0181	0.1543 30	0.7215 81	1.6401 98	2.6310	3.6301
64	0.0002	0.0019	0.0186	0.1573 30	0.7296 81	1.6498 98	2.6410	3.6401
65	0.0002	0.0019	0.0190	0.1604 31	0.7377 82	1.6596 98	2.6510	3.6501
66	0.0002	0.0020	0.0194	0.1635 31	0.7459 82	1.6694 98	2.6609	3.6601
67	0.0002	0.0020	0.0199	0.1666 32	0.7541 82	1.6792 98	2.6709	3.6701
68	0.0002	0.0021	0.0203	0.1699 32	0.7624 83	1.6890 98	2.6809	3.6801
69	0.0002	0.0021	0.0208	0.1731 33	0.7707 83	1.6988 98	2.6909	3.6901
70	0.0002	0.0022	0.0212	0.1764 33	0.7790 83	1.7086 98	2.7009	3.7001
71	0.0002	0.0022	0.0217	0.1798 34	0.7874 84	1.7184 98	2.7108	3.7101
72	0.0002	0.0023	0.0222	0.1832 34	0.7957 84	1.7282 98	2.7208	3.7201
73	0.0002	0.0023	0.0227	0.1867 35	0.8042 84	1.7380 98	2.7308	3.7301
74	0.0002	0.0024	0.0232	0.1902 35	0.8126 85	1.7478 98	2.7408	3.7401
75	0.0002	0.0024	0.0238	0.1938 36	0.8211 85	1.7577 98	2.7508	3.7501
76	0.0002	0.0025	0.0243	0.1974 37	0.8296 85	1.7675 98	2.7608	3.7601
77	0.0003	0.0025	0.0248	0.2011 37	0.8381 85	1.7773 98	2.7707	3.7701
78	0.0003	0.0026	0.0254	0.2048 38	0.8467 86	1.7871 98	2.7807	3.7801
79	0.0003	0.0027	0.0260	0.2086 38	0.8553 86	1.7970 98	2.7907	3.7901
80	0.0003	0.0027	0.0266	0.2124 39	0.8639 86	1.8068 98	2.8007	3.8001
81	0.0003	0.0028	0.0272	0.2163 39	0.8725 87	1.8167 98	2.8107	3.8101
82	0.0003	0.0029	0.0278	0.2203 40	0.8812 87	1.8265 99	2.8207	3.8201
83	0.0003	0.0029	0.0284	0.2243 40	0.8899 87	1.8364 99	2.8306	3.8301
84	0.0003	0.0030	0.0291	0.2284 41	0.8986 87	1.8462 99	2.8406	3.8401
85	0.0003	0.0031	0.0297	0.2325 41	0.9074 88	1.8561 99	2.8506	3.8501
86	0.0003	0.0031	0.0304	0.2366 42	0.9162 88	1.8660 99	2.8606	3.8601
87	0.0003	0.0032	0.0311	0.2409 43	0.9250 88	1.8758 99	2.8706	3.8701
88	0.0003	0.0033	0.0318	0.2452 43	0.9338 88	1.8857 99	2.8806	3.8801
89	0.0003	0.0034	0.0325	0.2495 44	0.9426 89	1.8956 99	2.8906	3.8901
90	0.0003	0.0034	0.0332	0.2539 44	0.9515 89	1.9054 99	2.9005	3.9001
91	0.0004	0.0035	0.0339	0.2584 45	0.9604 89	1.9153 99	2.9105	3.9101
92	0.0004	0.0036	0.0347	0.2629 45	0.9693 89	1.9252 99	2.9205	3.9201
93	0.0004	0.0037	0.0355	0.2674 46	0.9782 89	1.9351 99	2.9305	3.9301
94	0.0004	0.0038	0.0363	0.2721 47	0.9872 90	1.9450 99	2.9405	3.9400
95	0.0004	0.0039	0.0371	0.2767 47	0.9962 90	1.9548 99	2.9505	3.9500
96	0.0004	0.0039	0.0379	0.2815 48	1.0052 90	1.9647 99	2.9605	3.9600
97	0.0004	0.0040	0.0387	0.2863 48	1.0142 90	1.9746 99	2.9705	3.9700
98	0.0004	0.0041	0.0396	0.2911 49	1.0232 91	1.9845 99	2.9805	3.9800
99	0.0004	0.0042	0.0405	0.2961 49	1.0323 91	1.9944 99	2.9904	3.9900
00	0.0004	0.0043	0.0414	0.3010 50	1.0414 91	2.0043 99	3.0004	4.0000

Logarithms of Circular Functions.

ϕ	0°	lsin	ltan	lsc	1°	lsin	ltan	lsc	2°	lsin	ltan	lsc	
00'	—	∞		00	8.2419	19	72	01	8.5428	31	36	03	60'
01'	6.4637	37		00	8.2490	91	71	01	8.5484	67	36	03	59'
02'	6.7648	48		00	8.2561	62	70	01	8.5500	03	36	03	58'
03'	6.9408	08		00	8.2630	31	69	01	8.5535	38	35	03	57'
04'	7.0658	58		00	8.2699	*00	68	01	8.5571	73	35	03	56'
05'	7.1627	27		00	8.2766	67	67	01	8.5605	08	35	03	55'
06'	7.2419	19		00	8.2832	33	66	01	8.5640	43	34	03	54'
07'	7.3088	88		00	8.2898	99	65	01	8.5674	77	34	03	53'
08'	7.3668	68		00	8.2962	63	64	01	8.5708	11	34	03	52'
09'	7.4180	80		00	8.3026	26	63	01	8.5742	45	34	03	51'
10'	7.4637	37		00	8.3088	89	62	01	8.5776	79	33	03	50'
11'	7.5051	51		00	8.3150	50	61	01	8.5809	12	33	03	49'
12'	7.5429	29		00	8.3210	11	60	01	8.5842	45	33	03	48'
13'	7.5777	77		00	8.3270	71	59	01	8.5875	78	33	03	47'
14'	7.6099	99		00	8.3329	30	59	01	8.5907	11	32	03	46'
15'	7.6398	98		00	8.3388	89	58	01	8.5939	43	32	03	45'
16'	7.6678	78		00	8.3445	46	57	01	8.5972	75	32	03	44'
17'	7.6942	42		00	8.3502	03	56	01	8.6003	07	32	03	43'
18'	7.7190	90		00	8.3558	59	56	01	8.6035	38	31	04	42'
19'	7.7425	25		00	8.3613	14	55	01	8.6066	70	31	04	41'
20'	7.7648	48		00	8.3668	69	54	01	8.6097	*01	31	04	40'
21'	7.7859	60		00	8.3722	23	54	01	8.6128	32	31	04	39'
22'	7.8061	62		00	8.3775	76	53	01	8.6159	63	31	04	38'
23'	7.8255	55		00	8.3828	29	52	01	8.6189	93	30	04	37'
24'	7.8439	39		00	8.3880	81	52	01	8.6220	23	30	04	36'
25'	7.8617	17		00	8.3931	32	51	01	8.6250	54	30	04	35'
26'	7.8787	87		00	8.3982	83	51	01	8.6279	83	30	04	34'
27'	7.8951	51		00	8.4032	33	50	01	8.6309	13	30	04	33'
28'	7.9109	09		00	8.4082	83	49	01	8.6339	43	29	04	32'
29'	7.9261	61		00	8.4131	32	49	01	8.6368	72	29	04	31'
30'	7.9408	09		00	8.4179	81	48	01	8.6397	*01	29	04	30'
31'	7.9551	51		00	8.4227	29	48	02	8.6426	30	29	04	29'
32'	7.9689	89		00	8.4275	78	47	02	8.6454	59	29	04	28'
33'	7.9822	23	132	00	8.4322	23	47	02	8.6483	87	28	04	27'
34'	7.9952	52	128	00	8.4368	70	46	02	8.6511	15	28	04	26'
35'	8.0078	78	124	00	8.4414	16	46	02	8.6539	44	28	04	25'
36'	8.0200	00	121	00	8.4459	61	45	02	8.6567	71	28	04	24'
37'	8.0319	19	117	00	8.4504	06	45	02	8.6595	99	28	05	23'
38'	8.0435	35	114	00	8.4549	51	44	02	8.6622	27	27	05	22'
39'	8.0548	48	111	00	8.4593	95	44	02	8.6650	54	27	05	21'
40'	8.0658	58	109	00	8.4637	38	43	02	8.6677	82	27	05	20'
41'	8.0765	65	106	00	8.4680	82	43	02	8.6704	09	27	05	19'
42'	8.0870	70	103	00	8.4723	25	43	02	8.6731	36	27	05	18'
43'	8.0972	72	101	00	8.4765	67	42	02	8.6758	62	27	05	17'
44'	8.1072	72	99	00	8.4807	09	42	02	8.6784	89	26	05	16'
45'	8.1169	70	97	00	8.4848	51	41	02	8.6810	15	26	05	15'
46'	8.1265	65	94	00	8.4890	92	41	02	8.6837	42	26	05	14'
47'	8.1358	59	92	00	8.4930	33	41	02	8.6863	68	26	05	13'
48'	8.1450	50	90	00	8.4971	73	40	02	8.6889	94	26	05	12'
49'	8.1539	40	89	00	8.5011	13	40	02	8.6914	20	26	05	11'
50'	8.1627	27	87	00	8.5050	53	39	02	8.6940	45	26	05	10'
51'	8.1713	13	85	00	8.5090	92	39	02	8.6965	71	25	05	09'
52'	8.1797	98	84	00	8.5129	31	39	02	8.6991	96	25	05	08'
53'	8.1880	80	82	01	8.5167	70	38	02	8.7016	21	25	06	07'
54'	8.1961	62	80	01	8.5206	08	38	02	8.7041	46	25	06	06'
55'	8.2041	41	79	01	8.5243	46	38	02	8.7066	71	25	06	05'
56'	8.2119	20	78	01	8.5281	83	37	02	8.7090	96	25	06	04'
57'	8.2198	98	76	01	8.5318	21	37	03	8.7115	21	25	06	03'
58'	8.2271	72	75	01	8.5355	58	37	03	8.7140	45	24	06	02'
59'	8.2346	46	74	01	8.5392	94	37	03	8.7164	70	24	06	01'
60'	8.2419	19	72	01	8.5428	31	36	03	8.7188	94	24	06	00'
89° lcos lctn lsc 88° lcos lctn lsc 87° lcos lctn lsc θ													

Logarithms of Circular Functions.

ϕ	3°	lsin	ltn	lsc	4°	lsin	ltn	lsc	5°	lsin	ltn	lsc	
00'	8.7188	94	24	06	8.8436	46	18	11	8.9403	20	14	17	60'
01'	8.7212	18	24	06	8.8454	65	18	11	8.9417	34	14	17	59'
02'	8.7236	42	24	06	8.8472	83	18	11	8.9432	49	14	17	58'
03'	8.7260	66	24	06	8.8490	01	18	11	8.9446	63	14	17	57'
04'	8.7283	90	24	06	8.8508	18	18	11	8.9460	77	14	17	56'
05'	8.7307	13	23	06	8.8525	36	18	11	8.9475	92	14	17	55'
06'	8.7330	37	23	06	8.8543	54	18	11	8.9489	06	14	17	54'
07'	8.7354	60	23	06	8.8560	72	18	11	8.9503	20	14	17	53'
08'	8.7377	83	23	06	8.8578	89	18	11	8.9517	34	14	17	52'
09'	8.7400	06	23	07	8.8595	07	17	11	8.9531	49	14	18	51'
10'	8.7423	29	23	07	8.8613	24	17	11	8.9545	63	14	18	50'
11'	8.7445	52	23	07	8.8630	42	17	12	8.9559	77	14	18	49'
12'	8.7468	75	23	07	8.8647	59	17	12	8.9573	91	14	18	48'
13'	8.7491	97	23	07	8.8665	76	17	12	8.9587	06	14	18	47'
14'	8.7513	20	22	07	8.8682	94	17	12	8.9601	19	14	18	46'
15'	8.7535	42	22	07	8.8699	11	17	12	8.9614	33	14	18	45'
16'	8.7557	65	22	07	8.8716	28	17	12	8.9628	46	14	18	44'
17'	8.7580	87	22	07	8.8733	45	17	12	8.9642	60	14	18	43'
18'	8.7602	09	22	07	8.8749	62	17	12	8.9655	74	14	19	42'
19'	8.7623	31	22	07	8.8766	78	17	12	8.9669	88	14	19	41'
20'	8.7645	52	22	07	8.8783	95	17	12	8.9682	01	14	19	40'
21'	8.7667	74	22	07	8.8799	12	17	13	8.9696	15	14	19	39'
22'	8.7688	96	22	08	8.8816	29	17	13	8.9709	29	14	19	38'
23'	8.7710	17	21	08	8.8833	45	17	13	8.9723	42	13	19	37'
24'	8.7731	39	21	08	8.8849	62	16	13	8.9736	56	13	19	36'
25'	8.7752	60	21	08	8.8865	78	16	13	8.9750	69	13	19	35'
26'	8.7773	81	21	08	8.8882	95	16	13	8.9763	82	13	20	34'
27'	8.7794	02	21	08	8.8898	11	16	13	8.9776	96	13	20	33'
28'	8.7815	23	21	08	8.8914	27	16	13	8.9789	09	13	20	32'
29'	8.7836	44	21	08	8.8930	44	16	13	8.9803	23	13	20	31'
30'	8.7857	65	21	08	8.8946	60	16	13	8.9816	36	13	20	30'
31'	8.7877	86	21	08	8.8962	76	16	14	8.9829	49	13	20	29'
32'	8.7898	06	20	08	8.8978	92	16	14	8.9842	62	13	20	28'
33'	8.7918	27	20	08	8.8994	08	16	14	8.9855	75	13	20	27'
34'	8.7939	47	20	08	8.9010	24	16	14	8.9868	88	13	21	26'
35'	8.7959	67	20	08	8.9026	40	16	14	8.9881	01	13	21	25'
36'	8.7979	88	20	09	8.9042	56	16	14	8.9894	15	13	21	24'
37'	8.7999	08	20	09	8.9057	71	16	14	8.9907	28	13	21	23'
38'	8.8019	28	20	09	8.9073	87	16	14	8.9919	40	13	21	22'
39'	8.8039	48	20	09	8.9089	03	16	14	8.9932	53	13	21	21'
40'	8.8059	67	20	09	8.9104	18	16	14	8.9945	66	13	21	20'
41'	8.8078	87	20	09	8.9119	34	15	15	8.9958	79	13	21	19'
42'	8.8098	07	20	09	8.9135	50	15	15	8.9970	92	13	22	18'
43'	8.8117	26	19	09	8.9150	65	15	15	8.9983	06	13	22	17'
44'	8.8137	46	19	09	8.9166	80	15	15	8.9996	17	13	22	16'
45'	8.8156	65	19	09	8.9181	96	15	15	9.0008	30	13	22	15'
46'	8.8175	85	19	09	8.9196	11	15	15	9.0021	43	13	22	14'
47'	8.8194	04	19	09	8.9211	26	15	15	9.0033	56	13	22	13'
48'	8.8213	23	19	10	8.9226	41	15	15	9.0046	68	12	22	12'
49'	8.8232	42	19	10	8.9241	56	15	15	9.0058	80	12	22	11'
50'	8.8251	61	19	10	8.9256	72	15	15	9.0070	93	12	23	10'
51'	8.8270	80	19	10	8.9271	87	15	16	9.0083	05	12	23	09'
52'	8.8289	99	19	10	8.9286	02	15	16	9.0095	18	12	23	08'
53'	8.8307	17	19	10	8.9301	16	15	16	9.0107	30	12	23	07'
54'	8.8326	36	19	10	8.9315	31	15	16	9.0120	43	12	23	06'
55'	8.8345	55	19	10	8.9330	46	15	16	9.0132	55	12	23	05'
56'	8.8363	73	18	10	8.9345	61	15	16	9.0144	67	12	23	04'
57'	8.8381	92	18	10	8.9359	76	15	16	9.0156	80	12	23	03'
58'	8.8400	10	18	10	8.9374	90	15	16	9.0168	92	12	24	02'
59'	8.8418	28	18	11	8.9388	05	15	16	9.0180	04	12	24	01'
60'	8.8436	46	18	11	8.9403	20	14	17	9.0192	16	12	24	00'
	86°	lcos	lctn	lsec	85°	lcos	lctn	lsec	84°	lcos	lctn	lsec	0

Logarithms of Circular Functions.

ϕ	S	$\text{l sin } \phi$	ϕ	T	$\text{l tan } \phi$	ϕ	$\text{l sec } \phi$	ϕ	$\text{l sec } \phi$
6.46			6.46			0.00			0.00
0°00'.000	37	-∞	0°00'.000	37	-∞	0°00'.000	00	4°20'.701	13
1°51'.479	36	8.5108	0°44'.155	38	8.1087	0°52'.164	01	4°30'.818	14
2°49'.567	35	8.6929	1°40'.555	39	8.4663	1°30'.348	02	4°40'.762	15
3°32'.313	34	8.7904	2°15'.168	40	8.5948	1°56'.634	03	4°50'.271	16
4°07'.789	33	8.8574	2°42'.563	41	8.6751	2°17'.998	04	4°59'.477	17
4°38'.783	32	8.9085	3°05'.959	42	8.7336	2°36'.469	05	5°08'.407	18
5°06'.659	31	8.9498	3°26'.717	43	8.7796	2°52'.976	06	5°17'.084	19
5°32'.201	30	8.9845	3°45'.567	44	8.8176	3°08'.038	07	5°25'.528	20
5°55'.913	29	9.0143	4°02'.954	45	8.8500	3°21'.977	08	5°33'.758	21
6°18'.138		9.0405	4°19'.171	46	8.8781	3°35'.016	09	5°41'.789	22
			4°34'.427	47	8.9031	3°47'.300	10	5°49'.633	23
			4°48'.875	48	8.9255	3°58'.955	11	5°57'.305	24
			5°02'.628	49	8.9458	4°10'.064	12	6°04'.814	25
			5°15'.780	50	8.9643	4°20'.701		6°12'.170	
			5°28'.401	51	8.9815				
			5°40'.550	52	8.9973				
			5°52'.278	53	9.0121				
			6°03'.623		9.0260				

ϕ	$\text{l sin } \phi$	$\text{l csc } \phi$	$\text{l tan } \phi$	$\text{l ctg } \phi$	$\text{l sec } \phi$	$\text{l cos } \phi$
6° 00'	9.0192 120 0.9808	9.0216 122 0.9784	0.0024 1 9.9976	84° 00'		
6° 10'	9.0311 117 0.9689	9.0336 118 0.9664	0.0025 1 9.9975	83° 50'		
6° 20'	9.0426 114 0.9574	9.0453 115 0.9547	0.0027 1 9.9973	83° 40'		
6° 30'	9.0539 111 0.9461	9.0567 112 0.9433	0.0028 1 9.9972	83° 30'		
6° 40'	9.0648 108 0.9352	9.0678 110 0.9322	0.0029 1 9.9971	83° 20'		
6° 50'	9.0755 105 0.9245	9.0786 107 0.9214	0.0031 2 9.9969	83° 10'		
7° 00'	9.0859 103 0.9141	9.0891 104 0.9109	0.0032 2 9.9968	83° 00'		
7° 10'	9.0961 100 0.9039	9.0995 102 0.9005	0.0034 2 9.9966	82° 50'		
7° 20'	9.1060 98 0.8940	9.1096 100 0.8904	0.0036 2 9.9964	82° 40'		
7° 30'	9.1157 96 0.8843	9.1194 98 0.8806	0.0037 2 9.9963	82° 30'		
7° 40'	9.1252 94 0.8748	9.1291 96 0.8709	0.0039 2 9.9961	82° 20'		
7° 50'	9.1345 92 0.8655	9.1385 94 0.8615	0.0041 2 9.9959	82° 10'		
8° 00'	9.1436 90 0.8564	9.1478 92 0.8522	0.0042 2 9.9958	82° 00'		
8° 10'	9.1525 88 0.8475	9.1569 90 0.8431	0.0044 2 9.9956	81° 50'		
8° 20'	9.1612 86 0.8388	9.1658 88 0.8342	0.0046 2 9.9954	81° 40'		
8° 30'	9.1697 85 0.8303	9.1745 86 0.8255	0.0048 2 9.9952	81° 30'		
8° 40'	9.1781 83 0.8219	9.1831 85 0.8169	0.0050 2 9.9950	81° 20'		
8° 50'	9.1863 81 0.8137	9.1915 83 0.8085	0.0052 2 9.9948	81° 10'		
9° 00'	9.1943 80 0.8057	9.1997 82 0.8003	0.0054 2 9.9946	81° 00'		
9° 10'	9.2022 78 0.7978	9.2078 80 0.7922	0.0056 2 9.9944	80° 50'		
9° 20'	9.2100 77 0.7900	9.2158 79 0.7842	0.0058 2 9.9942	80° 40'		
9° 30'	9.2176 75 0.7824	9.2236 78 0.7764	0.0060 2 9.9940	80° 30'		
9° 40'	9.2251 74 0.7749	9.2313 76 0.7687	0.0062 2 9.9938	80° 20'		
9° 50'	9.2324 73 0.7676	9.2389 75 0.7611	0.0064 2 9.9936	80° 10'		
10° 00'	9.2397 72 0.7603	9.2463 74 0.7537	0.0066 2 9.9934	80° 00'		

$\text{l cos } \theta$ $\text{l sec } \theta$ $\text{l ctg } \theta$ $\text{l tan } \theta$ $\text{l csc } \theta$ $\text{l sin } \theta$ θ

Logarithms of Circular Functions.

ϕ	$\text{lsin } \phi$	$\text{lcsc } \phi$	$\text{ltan } \phi$	$\text{lctn } \phi$	$\text{lsec } \phi$	$\text{lcos } \phi$	
10° 00'	9.2397 72 0.7603	9.2463 74 0.7537	0.0066 2 9.9934				80° 00'
10° 10'	9.2468 70 0.7532	9.2536 73 0.7464	0.0069 2 9.9931				79° 50'
10° 20'	9.2538 69 0.7462	9.2609 72 0.7391	0.0071 2 9.9929				79° 40'
10° 30'	9.2606 68 0.7394	9.2680 71 0.7320	0.0073 2 9.9927				79° 30'
10° 40'	9.2674 67 0.7326	9.2750 69 0.7250	0.0076 2 9.9924				79° 20'
10° 50'	9.2740 66 0.7260	9.2819 68 0.7181	0.0078 2 9.9922				79° 10'
11° 00'	9.2806 65 0.7194	9.2887 67 0.7113	0.0081 2 9.9919				79° 00'
11° 10'	9.2870 64 0.7130	9.2953 66 0.7047	0.0083 2 9.9917				78° 50'
11° 20'	9.2934 63 0.7066	9.3020 66 0.6980	0.0086 3 9.9914				78° 40'
11° 30'	9.2997 62 0.7003	9.3085 65 0.6915	0.0088 3 9.9912				78° 30'
11° 40'	9.3058 61 0.6942	9.3149 64 0.6851	0.0091 3 9.9909				78° 20'
11° 50'	9.3119 60 0.6881	9.3212 63 0.6788	0.0093 3 9.9907				78° 10'
12° 00'	9.3179 59 0.6821	9.3275 62 0.6725	0.0096 3 9.9904				78° 00'
12° 10'	9.3238 59 0.6762	9.3336 61 0.6664	0.0099 3 9.9901				77° 50'
12° 20'	9.3296 58 0.6704	9.3397 61 0.6603	0.0101 3 9.9899				77° 40'
12° 30'	9.3353 57 0.6647	9.3458 60 0.6542	0.0104 3 9.9896				77° 30'
12° 40'	9.3410 56 0.6590	9.3517 59 0.6483	0.0107 3 9.9893				77° 20'
12° 50'	9.3466 55 0.6534	9.3576 58 0.6424	0.0110 3 9.9890				77° 10'
13° 00'	9.3521 55 0.6479	9.3634 58 0.6366	0.0113 3 9.9887				77° 00'
13° 10'	9.3575 54 0.6425	9.3691 57 0.6309	0.0116 3 9.9884				76° 50'
13° 20'	9.3629 53 0.6371	9.3748 56 0.6252	0.0119 3 9.9881				76° 40'
13° 30'	9.3682 53 0.6318	9.3804 56 0.6196	0.0122 3 9.9878				76° 30'
13° 40'	9.3734 52 0.6266	9.3859 55 0.6141	0.0125 3 9.9875				76° 20'
13° 50'	9.3786 51 0.6214	9.3914 54 0.6086	0.0128 3 9.9872				76° 10'
14° 00'	9.3837 51 0.6163	9.3968 54 0.6032	0.0131 3 9.9869				76° 00'
14° 10'	9.3887 50 0.6113	9.4021 53 0.5979	0.0134 3 9.9866				75° 50'
14° 20'	9.3937 49 0.6063	9.4074 53 0.5926	0.0137 3 9.9863				75° 40'
14° 30'	9.3986 49 0.6014	9.4127 52 0.5873	0.0141 3 9.9859				75° 30'
14° 40'	9.4035 48 0.5965	9.4178 52 0.5822	0.0144 3 9.9856				75° 20'
14° 50'	9.4083 48 0.5917	9.4230 51 0.5770	0.0147 3 9.9853				75° 10'
15° 00'	9.4130 47 0.5870	9.4281 51 0.5719	0.0151 3 9.9849				75° 00'
15° 10'	9.4177 47 0.5823	9.4331 50 0.5669	0.0154 3 9.9846				74° 50'
15° 20'	9.4223 46 0.5777	9.4381 50 0.5619	0.0157 3 9.9843				74° 40'
15° 30'	9.4269 46 0.5731	9.4430 49 0.5570	0.0161 4 9.9839				74° 30'
15° 40'	9.4314 45 0.5686	9.4479 49 0.5521	0.0164 4 9.9836				74° 20'
15° 50'	9.4359 45 0.5641	9.4527 48 0.5473	0.0168 4 9.9832				74° 10'
16° 00'	9.4403 44 0.5597	9.4575 48 0.5425	0.0172 4 9.9828				74° 00'
16° 10'	9.4447 44 0.5553	9.4622 47 0.5378	0.0175 4 9.9825				73° 50'
16° 20'	9.4491 43 0.5509	9.4669 47 0.5331	0.0179 4 9.9821				73° 40'
16° 30'	9.4533 43 0.5467	9.4716 46 0.5284	0.0183 4 9.9817				73° 30'
16° 40'	9.4576 42 0.5424	9.4762 46 0.5238	0.0186 4 9.9814				73° 20'
16° 50'	9.4618 42 0.5382	9.4808 46 0.5192	0.0190 4 9.9810				73° 10'
17° 00'	9.4659 41 0.5341	9.4853 45 0.5147	0.0194 4 9.9806				73° 00'
	$\text{lcos } \theta$	$\text{lsec } \theta$	$\text{lctn } \theta$	$\text{ltan } \theta$	$\text{lcsc } \theta$	$\text{lsin } \theta$	

Logarithms of Circular Functions.

ϕ	$\log \sin \phi$	$\log \csc \phi$	$\log \tan \phi$	$\log \cot \phi$	$\log \sec \phi$	$\log \cos \phi$	
17° 00'	9.4659 41	0.5341	9.4853 45	0.5147	0.0194 4	9.9806	73° 00'
17° 10'	9.4700 41	0.5300	9.4898 45	0.5102	0.0198 4	9.9802	72° 50'
17° 20'	9.4741 40	0.5259	9.4943 44	0.5057	0.0202 4	9.9798	72° 40'
17° 30'	9.4781 40	0.5219	9.4987 44	0.5013	0.0206 4	9.9794	72° 30'
17° 40'	9.4821 40	0.5179	9.5031 44	0.4969	0.0210 4	9.9790	72° 20'
17° 50'	9.4861 39	0.5139	9.5075 43	0.4925	0.0214 4	9.9786	72° 10'
18° 00'	9.4900 39	0.5100	9.5118 43	0.4882	0.0218 4	9.9782	72° 00'
18° 10'	9.4939 38	0.5061	9.5161 43	0.4839	0.0222 4	9.9778	71° 50'
18° 20'	9.4977 38	0.5023	9.5203 42	0.4797	0.0226 4	9.9774	71° 40'
18° 30'	9.5015 38	0.4985	9.5245 42	0.4755	0.0230 4	9.9770	71° 30'
18° 40'	9.5052 37	0.4948	9.5287 42	0.4713	0.0235 4	9.9765	71° 20'
18° 50'	9.5090 37	0.4910	9.5329 41	0.4671	0.0239 4	9.9761	71° 10'
19° 00'	9.5126 37	0.4874	9.5370 41	0.4630	0.0243 4	9.9757	71° 00'
19° 10'	9.5163 36	0.4837	9.5411 41	0.4589	0.0248 4	9.9752	70° 50'
19° 20'	9.5199 36	0.4801	9.5451 40	0.4549	0.0252 4	9.9748	70° 40'
19° 30'	9.5235 36	0.4765	9.5491 40	0.4509	0.0257 4	9.9743	70° 30'
19° 40'	9.5270 35	0.4730	9.5531 40	0.4469	0.0261 5	9.9739	70° 20'
19° 50'	9.5306 35	0.4694	9.5571 40	0.4429	0.0266 5	9.9734	70° 10'
20° 00'	9.5341 35	0.4659	9.5611 39	0.4389	0.0270 5	9.9730	70° 00'
20° 10'	9.5375 34	0.4625	9.5650 39	0.4350	0.0275 5	9.9725	69° 50'
20° 20'	9.5409 34	0.4591	9.5689 39	0.4311	0.0279 5	9.9721	69° 40'
20° 30'	9.5443 34	0.4557	9.5727 39	0.4273	0.0284 5	9.9716	69° 30'
20° 40'	9.5477 33	0.4523	9.5766 38	0.4234	0.0289 5	9.9711	69° 20'
20° 50'	9.5510 33	0.4490	9.5804 38	0.4196	0.0294 5	9.9706	69° 10'
21° 00'	9.5543 33	0.4457	9.5842 38	0.4158	0.0298 5	9.9702	69° 00'
21° 10'	9.5576 33	0.4424	9.5879 38	0.4121	0.0303 5	9.9697	68° 50'
21° 20'	9.5609 32	0.4391	9.5917 37	0.4083	0.0308 5	9.9692	68° 40'
21° 30'	9.5641 32	0.4359	9.5954 37	0.4046	0.0313 5	9.9687	68° 30'
21° 40'	9.5673 32	0.4327	9.5991 37	0.4009	0.0318 5	9.9682	68° 20'
21° 50'	9.5704 32	0.4296	9.6028 37	0.3972	0.0323 5	9.9677	68° 10'
22° 00'	9.5736 31	0.4264	9.6064 36	0.3936	0.0328 5	9.9672	68° 00'
22° 10'	9.5767 31	0.4233	9.6100 36	0.3900	0.0333 5	9.9667	67° 50'
22° 20'	9.5798 31	0.4202	9.6136 36	0.3864	0.0339 5	9.9661	67° 40'
22° 30'	9.5828 30	0.4172	9.6172 36	0.3828	0.0344 5	9.9656	67° 30'
22° 40'	9.5859 30	0.4141	9.6208 36	0.3792	0.0349 5	9.9651	67° 20'
22° 50'	9.5889 30	0.4111	9.6243 35	0.3757	0.0354 5	9.9646	67° 10'
23° 00'	9.5919 30	0.4081	9.6279 35	0.3721	0.0360 5	9.9640	67° 00'
23° 10'	9.5948 30	0.4052	9.6314 35	0.3686	0.0365 5	9.9635	66° 50'
23° 20'	9.5978 29	0.4022	9.6348 35	0.3652	0.0371 5	9.9629	66° 40'
23° 30'	9.6007 29	0.3993	9.6383 35	0.3617	0.0376 5	9.9624	66° 30'
23° 40'	9.6036 29	0.3964	9.6417 34	0.3583	0.0382 6	9.9618	66° 20'
23° 50'	9.6065 29	0.3935	9.6452 34	0.3548	0.0387 6	9.9613	66° 10'
24° 00'	9.6093 28	0.3907	9.6486 34	0.3514	0.0393 6	9.9607	66° 00'
	$\log \cos \theta$	$\log \sec \theta$	$\log \cot \theta$	$\log \tan \theta$	$\log \csc \theta$	$\log \sin \theta$	θ

Logarithms of Circular Functions.

ϕ	$\lg \sin \phi$	$\lg \csc \phi$	$\lg \tan \phi$	$\lg \cot \phi$	$\lg \sec \phi$	$\lg \cos \phi$	
24° 00'	9.6093 28	0.3907	9.6486 34	0.3514	0.0393 6	9.9607	66° 00'
24° 10'	9.6121 28	0.3879	9.6520 34	0.3480	0.0398 6	9.9602	65° 50'
24° 20'	9.6149 28	0.3851	9.6553 34	0.3447	0.0404 6	9.9596	65° 40'
24° 30'	9.6177 28	0.3823	9.6587 33	0.3413	0.0410 6	9.9590	65° 30'
24° 40'	9.6205 28	0.3795	9.6620 33	0.3380	0.0416 6	9.9584	65° 20'
24° 50'	9.6232 27	0.3768	9.6654 33	0.3346	0.0421 6	9.9579	65° 10'
25° 00'	9.6259 27	0.3741	9.6687 33	0.3313	0.0427 6	9.9573	65° 00'
25° 10'	9.6286 27	0.3714	9.6720 33	0.3280	0.0433 6	9.9567	64° 50'
25° 20'	9.6313 27	0.3687	9.6752 33	0.3248	0.0439 6	9.9561	64° 40'
25° 30'	9.6340 26	0.3660	9.6785 33	0.3215	0.0445 6	9.9555	64° 30'
25° 40'	9.6366 26	0.3634	9.6817 32	0.3183	0.0451 6	9.9549	64° 20'
25° 50'	9.6392 26	0.3608	9.6850 32	0.3150	0.0457 6	9.9543	64° 10'
26° 00'	9.6418 26	0.3582	9.6882 32	0.3118	0.0463 6	9.9537	64° 00'
26° 10'	9.6444 26	0.3556	9.6914 32	0.3086	0.0470 6	9.9530	63° 50'
26° 20'	9.6470 26	0.3530	9.6946 32	0.3054	0.0476 6	9.9524	63° 40'
26° 30'	9.6495 25	0.3505	9.6977 32	0.3023	0.0482 6	9.9518	63° 30'
26° 40'	9.6521 25	0.3479	9.7009 31	0.2991	0.0488 6	9.9512	63° 20'
26° 50'	9.6546 25	0.3454	9.7040 31	0.2960	0.0495 6	9.9505	63° 10'
27° 00'	9.6570 25	0.3430	9.7072 31	0.2928	0.0501 6	9.9499	63° 00'
27° 10'	9.6595 25	0.3405	9.7103 31	0.2897	0.0508 6	9.9492	62° 50'
27° 20'	9.6620 24	0.3380	9.7134 31	0.2866	0.0514 7	9.9486	62° 40'
27° 30'	9.6644 24	0.3356	9.7165 31	0.2835	0.0521 7	9.9479	62° 30'
27° 40'	9.6668 24	0.3332	9.7196 31	0.2804	0.0527 7	9.9473	62° 20'
27° 50'	9.6692 24	0.3308	9.7226 31	0.2774	0.0534 7	9.9466	62° 10'
28° 00'	9.6716 24	0.3284	9.7257 30	0.2743	0.0541 7	9.9459	62° 00'
28° 10'	9.6740 24	0.3260	9.7287 30	0.2713	0.0547 7	9.9453	61° 50'
28° 20'	9.6763 23	0.3237	9.7317 30	0.2683	0.0554 7	9.9446	61° 40'
28° 30'	9.6787 23	0.3213	9.7348 30	0.2652	0.0561 7	9.9439	61° 30'
28° 40'	9.6810 23	0.3190	9.7378 30	0.2622	0.0568 7	9.9432	61° 20'
28° 50'	9.6833 23	0.3167	9.7408 30	0.2592	0.0575 7	9.9425	61° 10'
29° 00'	9.6856 23	0.3144	9.7438 30	0.2562	0.0582 7	9.9418	61° 00'
29° 10'	9.6878 23	0.3122	9.7467 30	0.2533	0.0589 7	9.9411	60° 50'
29° 20'	9.6901 22	0.3099	9.7497 30	0.2503	0.0596 7	9.9404	60° 40'
29° 30'	9.6923 22	0.3077	9.7526 29	0.2474	0.0603 7	9.9397	60° 30'
29° 40'	9.6946 22	0.3054	9.7556 29	0.2444	0.0610 7	9.9390	60° 20'
29° 50'	9.6968 22	0.3032	9.7585 29	0.2415	0.0617 7	9.9383	60° 10'
30° 00'	9.6990 22	0.3010	9.7614 29	0.2386	0.0625 7	9.9375	60° 00'
30° 10'	9.7012 22	0.2988	9.7644 29	0.2356	0.0632 7	9.9368	59° 50'
30° 20'	9.7033 22	0.2967	9.7673 29	0.2327	0.0639 7	9.9361	59° 40'
30° 30'	9.7055 21	0.2945	9.7701 29	0.2299	0.0647 7	9.9353	59° 30'
30° 40'	9.7076 21	0.2924	9.7730 29	0.2270	0.0654 7	9.9346	59° 20'
30° 50'	9.7097 21	0.2903	9.7759 29	0.2241	0.0662 8	9.9338	59° 10'
31° 00'	9.7118 21	0.2882	9.7788 29	0.2212	0.0669 8	9.9331	59° 00'
	$\lg \cos \theta$	$\lg \sec \theta$	$\lg \cot \theta$	$\lg \tan \theta$	$\lg \csc \theta$	$\lg \sin \theta$	θ

Logarithms of Circular Functions.

ϕ	$\lg \sin \phi$	$\lg \csc \phi$	$\lg \tan \phi$	$\lg \cot \phi$	$\lg \sec \phi$	$\lg \cos \phi$	
31° 00'	9.7118 21 0.2882	9.7788 29 0.2212	0.0669 8 9.9331				59° 00'
31° 10'	9.7139 21 0.2861	9.7816 29 0.2184	0.0677 8 9.9323				58° 50'
31° 20'	9.7160 21 0.2840	9.7845 28 0.2155	0.0685 8 9.9315				58° 40'
31° 30'	9.7181 21 0.2819	9.7873 28 0.2127	0.0692 8 9.9308				58° 30'
31° 40'	9.7201 20 0.2799	9.7902 28 0.2098	0.0700 8 9.9300				58° 20'
31° 50'	9.7222 20 0.2778	9.7930 28 0.2070	0.0708 8 9.9292				58° 10'
32° 00'	9.7242 20 0.2758	9.7958 28 0.2042	0.0716 8 9.9284				58° 00'
32° 10'	9.7262 20 0.2738	9.7986 28 0.2014	0.0724 8 9.9276				57° 50'
32° 20'	9.7282 20 0.2718	9.8014 28 0.1986	0.0732 8 9.9268				57° 40'
32° 30'	9.7302 20 0.2698	9.8042 28 0.1958	0.0740 8 9.9260				57° 30'
32° 40'	9.7322 20 0.2678	9.8070 28 0.1930	0.0748 8 9.9252				57° 20'
32° 50'	9.7342 20 0.2658	9.8097 28 0.1903	0.0756 8 9.9244				57° 10'
33° 00'	9.7361 19 0.2639	9.8125 28 0.1875	0.0764 8 9.9236				57° 00'
33° 10'	9.7380 19 0.2620	9.8153 28 0.1847	0.0772 8 9.9228				56° 50'
33° 20'	9.7400 19 0.2600	9.8180 28 0.1820	0.0781 8 9.9219				56° 40'
33° 30'	9.7419 19 0.2581	9.8208 27 0.1792	0.0789 8 9.9211				56° 30'
33° 40'	9.7438 19 0.2562	9.8235 27 0.1765	0.0797 8 9.9203				56° 20'
33° 50'	9.7457 19 0.2543	9.8263 27 0.1737	0.0806 8 9.9194				56° 10'
34° 00'	9.7476 19 0.2524	9.8290 27 0.1710	0.0814 9 9.9186				56° 00'
34° 10'	9.7494 19 0.2506	9.8317 27 0.1683	0.0823 9 9.9177				55° 50'
34° 20'	9.7513 18 0.2487	9.8344 27 0.1656	0.0831 9 9.9169				55° 40'
34° 30'	9.7531 18 0.2469	9.8371 27 0.1629	0.0840 9 9.9160				55° 30'
34° 40'	9.7550 18 0.2450	9.8398 27 0.1602	0.0849 9 9.9151				55° 20'
34° 50'	9.7568 18 0.2432	9.8425 27 0.1575	0.0858 9 9.9142				55° 10'
35° 00'	9.7586 18 0.2414	9.8452 27 0.1548	0.0866 9 9.9134				55° 00'
35° 10'	9.7604 18 0.2396	9.8479 27 0.1521	0.0875 9 9.9125				54° 50'
35° 20'	9.7622 18 0.2378	9.8506 27 0.1494	0.0884 9 9.9116				54° 40'
35° 30'	9.7640 18 0.2360	9.8533 27 0.1467	0.0893 9 9.9107				54° 30'
35° 40'	9.7657 18 0.2343	9.8559 27 0.1441	0.0902 9 9.9098				54° 20'
35° 50'	9.7675 17 0.2325	9.8586 27 0.1414	0.0911 9 9.9089				54° 10'
36° 00'	9.7692 17 0.2308	9.8613 27 0.1387	0.0920 9 9.9080				54° 00'
36° 10'	9.7710 17 0.2290	9.8639 27 0.1361	0.0930 9 9.9070				53° 50'
36° 20'	9.7727 17 0.2273	9.8666 26 0.1334	0.0939 9 9.9061				53° 40'
36° 30'	9.7744 17 0.2256	9.8692 26 0.1308	0.0948 9 9.9052				53° 30'
36° 40'	9.7761 17 0.2239	9.8718 26 0.1282	0.0958 9 9.9042				53° 20'
36° 50'	9.7778 17 0.2222	9.8745 26 0.1255	0.0967 9 9.9033				53° 10'
37° 00'	9.7795 17 0.2205	9.8771 26 0.1229	0.0977 10 9.9023				53° 00'
37° 10'	9.7811 17 0.2189	9.8797 26 0.1203	0.0986 10 9.9014				52° 50'
37° 20'	9.7828 17 0.2172	9.8824 26 0.1176	0.0996 10 9.9004				52° 40'
37° 30'	9.7844 16 0.2156	9.8850 26 0.1150	0.1005 10 9.8995				52° 30'
37° 40'	9.7861 16 0.2139	9.8876 26 0.1124	0.1015 10 9.8985				52° 20'
37° 50'	9.7877 16 0.2123	9.8902 26 0.1098	0.1025 10 9.8975				52° 10'
38° 00'	9.7893 16 0.2107	9.8928 26 0.1072	0.1035 10 9.8965				52° 00'
$\lg \cos \theta$ $\lg \sec \theta$ $\lg \cot \theta$ $\lg \tan \theta$ $\lg \csc \theta$ $\lg \sin \theta$ θ							

Logarithms of Circular Functions.

ϕ	$\log \sin \phi$	$\log \csc \phi$	$\log \tan \phi$	$\log \cot \phi$	$\log \sec \phi$	$\log \cos \phi$	
38° 00'	9.7893 16	0.2107	9.8928 26	0.1072	0.1035 10	9.8965	52° 00'
38° 10'	9.7910 16	0.2090	9.8954 26	0.1046	0.1045 10	9.8955	51° 50'
38° 20'	9.7926 16	0.2074	9.8980 26	0.1020	0.1055 10	9.8945	51° 40'
38° 30'	9.7941 16	0.2059	9.9006 26	0.0994	0.1065 10	9.8935	51° 30'
38° 40'	9.7957 16	0.2043	9.9032 26	0.0968	0.1075 10	9.8925	51° 20'
38° 50'	9.7973 16	0.2027	9.9058 26	0.0942	0.1085 10	9.8915	51° 10'
39° 00'	9.7989 16	0.2011	9.9084 26	0.0916	0.1095 10	9.8905	51° 00'
39° 10'	9.8004 16	0.1996	9.9110 26	0.0890	0.1105 10	9.8895	50° 50'
39° 20'	9.8020 15	0.1980	9.9135 26	0.0865	0.1116 10	9.8884	50° 40'
39° 30'	9.8035 15	0.1965	9.9161 26	0.0839	0.1126 10	9.8874	50° 30'
39° 40'	9.8050 15	0.1950	9.9187 26	0.0813	0.1136 10	9.8864	50° 20'
39° 50'	9.8066 15	0.1934	9.9212 26	0.0788	0.1147 11	9.8853	50° 10'
40° 00'	9.8081 15	0.1919	9.9238 26	0.0762	0.1157 11	9.8843	50° 00'
40° 10'	9.8096 15	0.1904	9.9264 26	0.0736	0.1168 11	9.8832	49° 50'
40° 20'	9.8111 15	0.1889	9.9289 26	0.0711	0.1179 11	9.8821	49° 40'
40° 30'	9.8125 15	0.1875	9.9315 26	0.0685	0.1190 11	9.8810	49° 30'
40° 40'	9.8140 15	0.1860	9.9341 26	0.0659	0.1200 11	9.8800	49° 20'
40° 50'	9.8155 15	0.1845	9.9366 26	0.0634	0.1211 11	9.8789	49° 10'
41° 00'	9.8169 15	0.1831	9.9392 26	0.0608	0.1222 11	9.8778	49° 00'
41° 10'	9.8184 14	0.1816	9.9417 25	0.0583	0.1233 11	9.8767	48° 50'
41° 20'	9.8198 14	0.1802	9.9443 25	0.0557	0.1244 11	9.8756	48° 40'
41° 30'	9.8213 14	0.1787	9.9468 25	0.0532	0.1255 11	9.8745	48° 30'
41° 40'	9.8227 14	0.1773	9.9494 25	0.0506	0.1267 11	9.8733	48° 20'
41° 50'	9.8241 14	0.1759	9.9519 25	0.0481	0.1278 11	9.8722	48° 10'
42° 00'	9.8255 14	0.1745	9.9544 25	0.0456	0.1289 11	9.8711	48° 00'
42° 10'	9.8269 14	0.1731	9.9570 25	0.0430	0.1301 11	9.8699	47° 50'
42° 20'	9.8283 14	0.1717	9.9595 25	0.0405	0.1312 12	9.8688	47° 40'
42° 30'	9.8297 14	0.1703	9.9621 25	0.0379	0.1324 12	9.8676	47° 30'
42° 40'	9.8311 14	0.1689	9.9646 25	0.0354	0.1335 12	9.8665	47° 20'
42° 50'	9.8324 14	0.1676	9.9671 25	0.0329	0.1347 12	9.8653	47° 10'
43° 00'	9.8338 14	0.1662	9.9697 25	0.0303	0.1359 12	9.8641	47° 00'
43° 10'	9.8351 13	0.1649	9.9722 25	0.0278	0.1371 12	9.8629	46° 50'
43° 20'	9.8365 13	0.1635	9.9747 25	0.0253	0.1382 12	9.8618	46° 40'
43° 30'	9.8378 13	0.1622	9.9772 25	0.0228	0.1394 12	9.8606	46° 30'
43° 40'	9.8391 13	0.1609	9.9798 25	0.0202	0.1406 12	9.8594	46° 20'
43° 50'	9.8405 13	0.1595	9.9823 25	0.0177	0.1418 12	9.8582	46° 10'
44° 00'	9.8418 13	0.1582	9.9848 25	0.0152	0.1431 12	9.8569	46° 00'
44° 10'	9.8431 13	0.1569	9.9874 25	0.0126	0.1443 12	9.8557	45° 50'
44° 20'	9.8444 13	0.1556	9.9899 25	0.0101	0.1455 12	9.8545	45° 40'
44° 30'	9.8457 13	0.1543	9.9924 25	0.0076	0.1468 12	9.8532	45° 30'
44° 40'	9.8469 13	0.1531	9.9949 25	0.0051	0.1480 12	9.8520	45° 20'
44° 50'	9.8482 13	0.1518	9.9975 25	0.0025	0.1493 13	9.8507	45° 10'
45° 00'	9.8495 13	0.1505	0.0000 25	0.0000	0.1505 13	9.8495	45° 00'
	$\log \cos \theta$	$\log \sec \theta$	$\log \cot \theta$	$\log \tan \theta$	$\log \csc \theta$	$\log \sin \theta$	

Inverse Circular Functions.

log u sin ⁻¹ u cos ⁻¹ u tan ⁻¹ u ctn ⁻¹ u				log u sin ⁻¹ u cos ⁻¹ u tan ⁻¹ u ctn ⁻¹ u			
9.	°	°	°	9.	°	°	°
00	5.74 13 84.26	5.71 13 84.29	50	18.43 44 71.57	17.55 38 72.45		
01	5.87 14 84.13	5.84 13 84.16	51	18.88 45 71.12	17.93 39 72.07		
02	6.01 14 83.99	5.98 14 84.02	52	19.34 46 70.66	18.32 39 71.68		
03	6.15 14 83.85	6.12 14 83.88	53	19.81 48 70.19	18.72 40 71.28		
04	6.30 15 83.70	6.26 14 83.74	54	20.29 49 69.71	19.12 41 70.88		
05	6.44 15 83.56	6.40 15 83.60	55	20.78 50 69.22	19.54 42 70.46		
06	6.59 15 83.41	6.55 15 83.45	56	21.29 51 68.71	19.95 42 70.05		
07	6.75 16 83.25	6.70 15 83.30	57	21.81 53 68.19	20.38 43 69.62		
08	6.91 16 83.09	6.86 16 83.14	58	22.35 54 67.65	20.82 44 69.18		
09	7.07 16 82.93	7.01 16 82.99	59	22.90 56 67.10	21.26 45 68.74		
10	7.23 17 82.77	7.18 16 82.82	60	23.46 57 66.54	21.71 45 68.29		
11	7.40 17 82.60	7.34 17 82.66	61	24.04 59 65.96	22.17 46 67.83		
12	7.58 18 82.42	7.51 17 82.49	62	24.64 61 65.36	22.63 47 67.37		
13	7.75 18 82.25	7.68 17 82.32	63	25.25 62 64.75	23.10 48 66.90		
14	7.93 18 82.07	7.86 18 82.14	64	25.88 64 64.12	23.58 48 66.42		
15	8.12 19 81.88	8.04 18 81.96	65	26.53 66 63.47	24.07 49 65.93		
16	8.31 19 81.69	8.22 19 81.78	66	27.20 68 62.80	24.56 50 65.44		
17	8.51 20 81.49	8.41 19 81.59	67	27.89 70 62.11	25.07 51 64.93		
18	8.71 20 81.29	8.61 20 81.39	68	28.60 72 61.40	25.58 51 64.42		
19	8.91 21 81.09	8.80 20 81.20	69	29.33 74 60.67	26.09 52 63.91		
20	9.12 21 80.88	9.01 20 80.99	70	30.08 76 59.92	26.62 53 63.38		
21	9.33 22 80.67	9.21 21 80.79	71	30.85 79 59.15	27.15 54 62.85		
22	9.55 22 80.45	9.42 21 80.58	72	31.66 81 58.34	27.69 54 62.31		
23	9.78 23 80.22	9.64 22 80.36	73	32.48 84 57.52	28.24 55 61.76		
24	10.01 23 79.99	9.86 22 80.14	74	33.34 87 56.66	28.79 56 61.21		
25	10.24 24 79.76	10.08 23 79.92	75	34.22 90 55.78	29.35 56 60.65		
26	10.48 24 79.52	10.31 23 79.69	76	35.13 54.87	29.92 57 60.08		
27	10.73 25 79.27	10.55 24 79.45	77	36.07 53.93	30.49 58 59.51		
28	10.98 26 79.02	10.79 24 79.21	78	37.05 52.95	31.07 58 58.93		
29	11.24 26 78.76	11.03 25 78.97	79	38.07 51.93	31.66 59 58.34		
30	11.51 27 78.49	11.28 25 78.72	80	39.12 50.88	32.25 60 57.75		
31	11.78 28 78.22	11.54 26 78.46	81	40.21 49.79	32.85 60 57.15		
32	12.06 28 77.94	11.80 26 78.20	82	41.35 48.65	33.45 61 56.55		
33	12.34 29 77.66	12.07 27 77.93	83	42.54 47.46	34.06 61 55.94		
34	12.64 30 77.36	12.34 28 77.66	84	43.78 46.22	34.68 62 55.32		
35	12.94 30 77.06	12.62 28 77.38	85	45.07 44.93	35.30 62 54.70		
36	13.24 31 76.76	12.90 29 77.10	86	46.42 43.58	35.92 63 54.08		
37	13.56 32 76.44	13.19 29 76.81	87	47.84 42.16	36.55 63 53.45		
38	13.88 33 76.12	13.49 30 76.51	88	49.34 40.66	37.18 64 52.82		
39	14.21 33 75.79	13.79 31 76.21	89	50.92 39.08	37.82 64 52.18		
40	14.55 34 75.45	14.10 31 75.90	90	52.59 37.41	38.46 64 51.54		
41	14.89 35 75.11	14.42 32 75.58	91	54.37 35.63	39.11 65 50.89		
42	15.25 36 74.75	14.74 32 75.26	92	56.28 33.72	39.75 65 50.25		
43	15.61 37 74.39	15.06 33 74.94	93	58.34 31.66	40.40 65 49.60		
44	15.99 38 74.01	15.40 34 74.60	94	60.57 29.43	41.05 65 48.95		
45	16.37 39 73.63	15.74 34 74.26	95	63.03 26.97	41.71 66 48.29		
46	16.76 40 73.24	16.09 35 73.91	96	65.78 24.22	42.37 66 47.63		
47	17.16 41 72.84	16.44 36 73.56	97	68.95 21.05	43.02 66 46.98		
48	17.58 42 72.42	16.80 37 73.20	98	72.74 17.26	43.68 66 46.32		
49	18.00 43 72.00	17.17 37 72.83	99	77.75 12.25	44.34 66 45.66		
50	18.43 44 71.57	17.55 38 72.45	00	90.00 00.00	45.00 66 45.00		

Inverse Circular Functions.

log u	sin ⁻¹ u	cos ⁻¹ u	log u	sin ⁻¹ u	cos ⁻¹ u	log u	sin ⁻¹ u	cos ⁻¹ u
9.	°	°	9.	°	°	9.	°	°
750	34.22	9 55.78	800	39.12	11 50.88	850	45.07	13 44.93
751	34.31	9 55.89	801	39.23	11 50.77	851	45.20	13 44.80
752	34.40	9 55.60	802	39.34	11 50.66	852	45.33	13 44.67
753	34.49	9 55.51	803	39.44	11 50.56	853	45.47	13 44.53
754	34.58	9 55.42	804	39.55	11 50.45	854	45.60	13 44.40
755	34.67	9 55.33	805	39.66	11 50.34	855	45.74	14 44.26
756	34.76	9 55.24	806	39.77	11 50.23	856	45.87	14 44.13
757	34.85	9 55.15	807	39.88	11 50.12	857	46.01	14 43.99
758	34.95	9 55.05	808	39.99	11 50.01	858	46.15	14 43.85
759	35.04	9 54.96	809	40.10	11 49.90	859	46.28	14 43.72
760	35.13	9 54.87	810	40.21	11 49.79	860	46.42	14 43.58
761	35.22	9 54.78	811	40.33	11 49.67	861	46.56	14 43.44
762	35.32	9 54.68	812	40.44	11 49.56	862	46.70	14 43.30
763	35.41	9 54.59	813	40.55	11 49.45	863	46.84	14 43.16
764	35.50	9 54.50	814	40.66	11 49.34	864	46.98	14 43.02
765	35.60	9 54.40	815	40.78	11 49.22	865	47.12	14 42.88
766	35.69	9 54.31	816	40.89	11 49.11	866	47.27	14 42.73
767	35.79	10 54.21	817	41.01	11 48.99	867	47.41	14 42.59
768	35.88	10 54.12	818	41.12	12 48.88	868	47.55	14 42.45
769	35.98	10 54.02	819	41.24	12 48.76	869	47.70	14 42.30
770	36.07	10 53.93	820	41.35	12 48.65	870	47.84	15 42.16
771	36.17	10 53.83	821	41.47	12 48.53	871	47.99	15 42.01
772	36.27	10 53.73	822	41.59	12 48.41	872	48.14	15 41.86
773	36.36	10 53.64	823	41.70	12 48.30	873	48.28	15 41.72
774	36.46	10 53.54	824	41.82	12 48.18	874	48.43	15 41.57
775	36.56	10 53.44	825	41.94	12 48.06	875	48.58	15 41.42
776	36.66	10 53.34	826	42.06	12 47.94	876	48.73	15 41.27
777	36.76	10 53.24	827	42.18	12 47.82	877	48.88	15 41.12
778	36.85	10 53.15	828	42.30	12 47.70	878	49.03	15 40.97
779	36.95	10 53.05	829	42.42	12 47.58	879	49.19	15 40.81
780	37.05	10 52.95	830	42.54	12 47.46	880	49.34	15 40.66
781	37.15	10 52.85	831	42.66	12 47.34	881	49.49	15 40.51
782	37.25	10 52.75	832	42.78	12 47.22	882	49.65	16 40.35
783	37.35	10 52.65	833	42.90	12 47.10	883	49.80	16 40.20
784	37.45	10 52.55	834	43.03	12 46.97	884	49.96	16 40.04
785	37.56	10 52.44	835	43.15	12 46.85	885	50.12	16 39.88
786	37.66	10 52.34	836	43.27	12 46.73	886	50.28	16 39.72
787	37.76	10 52.24	837	43.40	12 46.60	887	50.44	16 39.56
788	37.86	10 52.14	838	43.52	13 46.48	888	50.60	16 39.40
789	37.96	10 52.04	839	43.65	13 46.35	889	50.76	16 39.24
790	38.07	10 51.93	840	43.78	13 46.22	890	50.92	16 39.08
791	38.17	10 51.83	841	43.90	13 46.10	891	51.08	16 38.92
792	38.28	10 51.72	842	44.03	13 45.97	892	51.25	16 38.76
793	38.38	10 51.62	843	44.16	13 45.84	893	51.41	17 38.59
794	38.48	10 51.52	844	44.29	13 45.71	894	51.58	17 38.42
795	38.59	11 51.41	845	44.41	13 45.59	895	51.74	17 38.26
796	38.69	11 51.31	846	44.54	13 45.46	896	51.91	17 38.09
797	38.80	11 51.20	847	44.67	13 45.33	897	52.08	17 37.92
798	38.91	11 51.09	848	44.80	13 45.20	898	52.25	17 37.75
799	39.01	11 50.99	849	44.94	13 45.06	899	52.42	17 37.58
800	39.12	11 50.88	850	45.07	13 44.93	900	52.59	17 37.41

Inverse Circular Functions.

log u sin ⁻¹ u cos ⁻¹ u			log u sin ⁻¹ u cos ⁻¹ u			log u sin ⁻¹ u		log u sin ⁻¹ u	
9.	°	°	9.	°	°	9.	°	9.	°
900	52.50	17 37.41	950	63.03	26 26.97	9900	77.75	9950	81.32
901	52.76	17 37.24	951	63.29	26 26.71	9901	77.81	9951	81.41
902	52.94	17 37.06	952	63.56	27 26.44	9902	77.87	9952	81.50
903	53.11	18 36.89	953	63.82	27 26.18	9903	77.94	9953	81.59
904	53.29	18 36.71	954	64.09	27 25.91	9904	78.00	9954	81.68
905	53.47	18 36.53	955	64.37	27 25.63	9905	78.06	9955	81.77
906	53.65	18 36.35	956	64.64	28 25.36	9906	78.12	9956	81.86
907	53.83	18 36.17	957	64.92	28 25.08	9907	78.18	9957	81.95
908	54.01	18 35.99	958	65.21	29 24.79	9908	78.25	9958	82.04
909	54.19	18 35.81	959	65.49	29 24.51	9909	78.31	9959	82.14
910	54.37	18 35.63	960	65.78	29 24.22	9910	78.38	9960	82.24
911	54.56	19 35.44	961	66.08	30 23.92	9911	78.44	9961	82.33
912	54.74	19 35.26	962	66.38	30 23.62	9912	78.50	9962	82.43
913	54.93	19 35.07	963	66.68	31 23.32	9913	78.57	9963	82.5
914	55.12	19 34.88	964	66.99	31 23.01	9914	78.64	9964	82.6
915	55.31	19 34.69	965	67.30	32 22.70	9915	78.70	9965	82.7
916	55.50	19 34.50	966	67.62	32 22.38	9916	78.77	9966	82.8
917	55.69	19 34.31	967	67.95	33 22.05	9917	78.83	9967	82.9
918	55.89	19 34.11	968	68.27	33 21.73	9918	78.90	9968	83.1
919	56.08	20 33.92	969	68.61	34 21.39	9919	78.97	9969	83.2
920	56.28	20 33.72	970	68.95	34 21.05	9920	79.04	9970	83.3
921	56.48	20 33.52	971	69.29	35 20.71	9921	79.10	9971	83.4
922	56.68	20 33.32	972	69.65	36 20.35	9922	79.17	9972	83.5
923	56.88	20 33.12	973	70.01	36 19.99	9923	79.24	9973	83.6
924	57.08	20 32.92	974	70.37	37 19.63	9924	79.31	9974	83.7
925	57.29	21 32.71	975	70.75	38 19.25	9925	79.38	9975	83.9
926	57.49	21 32.51	976	71.13	39 18.87	9926	79.45	9976	84.0
927	57.70	21 32.30	977	71.52	39 18.48	9927	79.52	9977	84.1
928	57.91	21 32.09	978	71.92	40 18.08	9928	79.60	9978	84.2
929	58.12	21 31.88	979	72.33	41 17.67	9929	79.67	9979	84.4
930	58.34	21 31.66	980	72.74	42 17.26	9930	79.74	9980	84.5
931	58.55	22 31.45	981	73.18	44 16.82	9931	79.81	9981	84.6
932	58.77	22 31.23	982	73.63	45 16.38	9932	79.89	9982	84.8
933	58.99	22 31.01	983	74.01	46 15.93	9933	79.96	9983	84.9
934	59.21	22 30.79	984	74.54	48 15.46	9934	80.04	9984	85.1
935	59.43	22 30.57	985	75.03	49 14.97	9935	80.11	9985	85.2
936	59.65	23 30.35	986	75.53	51 14.47	9936	80.19	9986	85.4
937	59.88	23 30.12	987	76.05	53 13.95	9937	80.26	9987	85.6
938	60.11	23 29.89	988	76.59	55 13.41	9938	80.34	9988	85.7
939	60.34	23 29.66	989	77.16	58 12.84	9939	80.42	9989	85.9
940	60.57	23 29.43	990	77.75	61 12.25	9940	80.50	9990	86.1
941	60.81	24 29.19	991	78.38	11 62	9941	80.58	9991	86.3
942	61.04	24 28.96	992	79.04	10.96	9942	80.66	9992	86.5
943	61.28	24 28.72	993	79.74	10.26	9943	80.74	9993	86.7
944	61.52	24 28.48	994	80.50	9.50	9944	80.82	9994	87.0
945	61.77	25 28.23	995	81.32	8.68	9945	80.90	9995	87.3
946	62.02	25 27.98	996	82.24	7.76	9946	80.98	9996	87.5
947	62.27	25 27.73	997	83.3	6.7	9947	81.07	9997	87.9
948	62.52	25 27.48	998	84.5	5.7	9948	81.15	9998	88.3
949	62.77	26 27.23	999	86.1	3.5	9949	81.24	9999	88.8
950	63.03	26 26.97	000	90.	0.	9950	81.32	0000	90.

Logarithms of Hyperbolic Functions.

x gd x		x gd x		x l Sh x l Ch x l Th x			
	°		°		0.	0.	9.
0.00	0.0000 5730	0.50	27.524 508	1.00	0701 57	1884 33	8817 24
0.01	0.5729 5729	0.51	28.031 506	1.01	0758 57	1917 33	8840 23
0.02	1.1458 5728	0.52	28.535 503	1.02	0815 56	1950 33	8864 23
0.03	1.7186 5727	0.53	29.037 501	1.03	0871 56	1984 34	8887 23
0.04	2.2912 5725	0.54	29.537 498	1.04	0927 56	2018 34	8909 22
0.05	2.8638 5722	0.55	30.034 496	1.05	0982 56	2051 34	8931 22
0.06	3.4367 5719	0.56	30.529 494	1.06	1038 55	2086 34	8952 21
0.07	4.0074 5716	0.57	31.021 491	1.07	1093 55	2120 34	8973 21
0.08	4.5788 5711	0.58	31.511 488	1.08	1148 55	2154 34	8994 20
0.09	5.1497 5706	0.59	31.998 486	1.09	1203 54	2189 35	9014 20
0.10	5.720 570	0.60	32.483 483	1.10	1257 54	2223 35	9034 19
0.11	6.290 570	0.61	32.965 481	1.11	1311 54	2258 35	9053 19
0.12	6.859 569	0.62	33.444 478	1.12	1365 54	2293 35	9072 19
0.13	7.428 568	0.63	33.921 475	1.13	1419 54	2328 35	9090 18
0.14	7.995 567	0.64	34.395 473	1.14	1472 53	2364 35	9108 18
0.15	8.562 567	0.65	34.867 470	1.15	1525 53	2399 36	9126 18
0.16	9.128 566	0.66	35.336 467	1.16	1578 53	2435 36	9144 17
0.17	9.694 565	0.67	35.802 465	1.17	1631 53	2470 36	9161 17
0.18	10.258 564	0.68	36.265 462	1.18	1684 52	2506 36	9177 17
0.19	10.821 563	0.69	36.726 459	1.19	1736 52	2542 36	9194 16
0.20	11.384 562	0.70	37.183 456	1.20	1788 52	2578 36	9210 16
0.21	11.945 561	0.71	37.638 454	1.21	1840 52	2615 36	9226 16
0.22	12.505 559	0.72	38.091 451	1.22	1892 52	2651 36	9241 15
0.23	13.063 558	0.73	38.540 448	1.23	1944 52	2688 37	9256 15
0.24	13.621 557	0.74	38.987 445	1.24	1995 51	2724 37	9271 15
0.25	14.177 556	0.75	39.431 443	1.25	2046 51	2761 37	9285 14
0.26	14.732 554	0.76	39.872 440	1.26	2098 51	2798 37	9300 14
0.27	15.285 553	0.77	40.310 437	1.27	2148 51	2835 37	9314 14
0.28	15.837 551	0.78	40.746 434	1.28	2199 51	2872 37	9327 14
0.29	16.388 550	0.79	41.179 431	1.29	2250 51	2909 37	9341 13
0.30	16.937 548	0.80	41.608 428	1.30	2300 50	2947 37	9354 13
0.31	17.484 546	0.81	42.035 426	1.31	2351 50	2984 38	9367 13
0.32	18.030 545	0.82	42.460 423	1.32	2401 50	3022 38	9379 12
0.33	18.573 543	0.83	42.881 420	1.33	2451 50	3059 38	9391 12
0.34	19.116 541	0.84	43.299 417	1.34	2501 50	3097 38	9404 12
0.35	19.656 540	0.85	43.715 414	1.35	2551 50	3135 38	9415 12
0.36	20.195 538	0.86	44.128 411	1.36	2600 50	3173 38	9427 11
0.37	20.732 536	0.87	44.537 408	1.37	2650 49	3211 38	9438 11
0.38	21.267 534	0.88	44.944 406	1.38	2699 49	3249 38	9450 11
0.39	21.800 532	0.89	45.348 403	1.39	2748 49	3288 38	9460 11
0.40	22.331 530	0.90	45.750 400	1.40	2797 49	3326 38	9471 11
0.41	22.859 528	0.91	46.148 397	1.41	2846 49	3365 39	9482 10
0.42	23.386 526	0.92	46.544 394	1.42	2895 49	3403 39	9492 10
0.43	23.911 524	0.93	46.936 391	1.43	2944 49	3442 39	9502 10
0.44	24.434 522	0.94	47.326 388	1.44	2993 49	3481 39	9512 10
0.45	24.955 519	0.95	47.713 386	1.45	3041 48	3520 39	9522 10
0.46	25.473 517	0.96	48.097 383	1.46	3090 48	3559 39	9531 9
0.47	25.989 515	0.97	48.478 380	1.47	3138 48	3598 39	9540 9
0.48	26.503 513	0.98	48.857 377	1.48	3186 48	3637 39	9549 9
0.49	27.015 510	0.99	49.232 374	1.49	3234 48	3676 39	9558 9
0.50	27.524 508	1.00	49.605 371	1.50	3282 48	3715 39	9567 9

Logarithms of Hyperbolic Functions.

x	$\text{Sh } x$	$\text{Ch } x$	$\text{Th } x$	x	$\text{Sh } x$	$\text{Ch } x$	$\text{Th } x$
	0.	0.	9.		0.	0.	9.
1.50	3282 48	3715 39	9567 9	2.00	5595 45	5754 42	9841 3
1.51	3330 48	3754 39	9578 8	2.01	5640 45	5796 42	9844 3
1.52	3378 48	3794 39	9584 8	2.02	5685 45	5838 42	9847 3
1.53	3426 48	3833 40	9592 8	2.03	5730 45	5880 42	9850 3
1.54	3474 48	3873 40	9601 8	2.04	5775 45	5922 42	9853 3
1.55	3521 48	3913 40	9608 8	2.05	5820 45	5964 42	9856 3
1.56	3569 47	3952 40	9616 8	2.06	5865 45	6006 42	9859 3
1.57	3616 47	3992 40	9624 8	2.07	5910 45	6048 42	9862 3
1.58	3663 47	4032 40	9631 7	2.08	5955 45	6090 42	9864 3
1.59	3711 47	4072 40	9639 7	2.09	6000 45	6132 42	9867 3
1.60	3758 47	4112 40	9646 7	2.10	6044 45	6175 42	9870 3
1.61	3805 47	4152 40	9653 7	2.11	6089 45	6217 42	9872 3
1.62	3852 47	4192 40	9660 7	2.12	6134 45	6259 42	9875 3
1.63	3899 47	4232 40	9666 7	2.13	6178 45	6301 42	9877 2
1.64	3946 47	4273 40	9673 7	2.14	6223 45	6343 42	9880 2
1.65	3992 47	4313 40	9679 6	2.15	6268 45	6386 42	9882 2
1.66	4039 47	4353 40	9686 6	2.16	6312 45	6428 42	9884 2
1.67	4086 47	4394 40	9692 6	2.17	6357 45	6470 42	9887 2
1.68	4132 47	4434 41	9698 6	2.18	6401 45	6512 42	9889 2
1.69	4179 46	4475 41	9704 6	2.19	6446 45	6555 42	9891 2
1.70	4225 46	4515 41	9710 6	2.20	6491 45	6597 42	9893 2
1.71	4272 46	4556 41	9716 6	2.21	6535 44	6640 42	9895 2
1.72	4318 46	4597 41	9721 6	2.22	6580 44	6682 42	9898 2
1.73	4364 46	4637 41	9727 5	2.23	6624 44	6724 42	9900 2
1.74	4411 46	4678 41	9732 5	2.24	6668 44	6767 42	9902 2
1.75	4457 46	4719 41	9738 5	2.25	6713 44	6809 42	9904 2
1.76	4503 46	4760 41	9743 5	2.26	6757 44	6852 42	9905 2
1.77	4549 46	4801 41	9748 5	2.27	6802 44	6894 43	9907 2
1.78	4595 46	4842 41	9753 5	2.28	6846 44	6937 43	9909 2
1.79	4641 46	4883 41	9758 5	2.29	6890 44	6979 43	9911 2
1.80	4687 46	4924 41	9763 5	2.30	6935 44	7022 43	9913 2
1.81	4733 46	4965 41	9767 5	2.31	6979 44	7064 43	9914 2
1.82	4778 46	5006 41	9772 5	2.32	7023 44	7107 43	9916 2
1.83	4824 46	5048 41	9776 4	2.33	7067 44	7150 43	9918 2
1.84	4870 46	5089 41	9781 4	2.34	7112 44	7192 43	9919 2
1.85	4915 46	5130 41	9785 4	2.35	7156 44	7235 43	9921 2
1.86	4961 46	5172 41	9789 4	2.36	7200 44	7278 43	9923 2
1.87	5007 46	5213 41	9794 4	2.37	7244 44	7320 43	9924 2
1.88	5052 46	5254 41	9798 4	2.38	7289 44	7363 43	9926 1
1.89	5098 45	5296 41	9802 4	2.39	7333 44	7406 43	9927 1
1.90	5143 45	5337 42	9806 4	2.40	7377 44	7448 43	9929 1
1.91	5188 45	5379 42	9810 4	2.41	7421 44	7491 43	9930 1
1.92	5234 45	5421 42	9813 4	2.42	7465 44	7534 43	9931 1
1.93	5279 45	5462 42	9817 4	2.43	7509 44	7577 43	9933 1
1.94	5324 45	5504 42	9821 4	2.44	7553 44	7619 43	9934 1
1.95	5370 45	5545 42	9824 4	2.45	7597 44	7662 43	9935 1
1.96	5415 45	5587 42	9828 3	2.46	7642 44	7705 43	9937 1
1.97	5460 45	5629 42	9831 3	2.47	7686 44	7748 43	9938 1
1.98	5505 45	5671 42	9834 3	2.48	7730 44	7791 43	9939 1
1.99	5550 45	5713 42	9838 3	2.49	7774 44	7833 43	9940 1
2.00	5595 45	5754 42	9841 3	2.50	7818 44	7876 43	9941 1

Logarithms of Hyperbolic Functions.

x	l Sh x	l Ch x	l Th x	x	l Sh x	l Ch x	l Th x																																												
2.50	0. 7818 44	0. 7876 43	9. 9941	3.0	1.0008 436	1.0029 432	9.9978																																												
2.51	7862 44	7919 43	9943	3.1	1.0444 436	1.0462 433	9.9982																																												
2.52	7908 44	7962 43	9944	3.2	1.0880 436	1.0894 433	9.9986																																												
2.53	7950 44	8005 43	9945	3.3	1.1316 435	1.1327 433	9.9988																																												
2.54	7994 44	8048 43	9946	3.4	1.1751 435	1.1761 433	9.9990																																												
2.55	8038 44	8091 43	9947	3.5	1.2186 435	1.2194 434	9.9992																																												
2.56	8082 44	8134 43	9948	3.6	1.2621 435	1.2628 434	9.9994																																												
2.57	8126 44	8176 43	9949	3.7	1.3056 435	1.3061 434	9.9995																																												
2.58	8169 44	8219 43	9950	3.8	1.3491 435	1.3495 434	9.9996																																												
2.59	8213 44	8262 43	9951	3.9	1.3925 435	1.3929 434	9.9996																																												
2.60	8257 44	8305 43	9952	4.0	1.4360 435	1.4363 434	9.9997																																												
2.61	8301 44	8348 43	9953	4.1	1.4795 435	1.4797 434	9.9998																																												
2.62	8345 44	8391 43	9954	4.2	1.5229 434	1.5231 434	9.9998																																												
2.63	8389 44	8434 43	9955	4.3	1.5664 434	1.5665 434	9.9998																																												
2.64	8433 44	8477 43	9956	4.4	1.6098 434	1.6099 434	9.9999																																												
2.65	8477 44	8520 43	9957	4.5	1.6532 434	1.6533 434	9.9999																																												
2.66	8521 44	8563 43	9958	4.6	1.6967 434	1.6968 434	9.9999																																												
2.67	8564 44	8606 43	9958	4.7	1.7401 434	1.7402 434	9.9999																																												
2.68	8608 44	8649 43	9959	4.8	1.7836 434	1.7836 434	9.9999																																												
2.69	8652 44	8692 43	9960	4.9	1.8270 434	1.8270 434	0.0000																																												
2.70	8696 44	8735 43	9961	5.0	1.8704 434	1.8705 434	0.0000																																												
2.71	8740 44	8778 43	9962	5.1	1.9139 434	1.9139 434	0.0000																																												
2.72	8784 44	8821 43	9962	5.2	1.9573 434	1.9573 434	0.0000																																												
2.73	8827 44	8864 43	9963	5.3	2.0007 434	2.0007 434	0.0000																																												
2.74	8871 44	8907 43	9964	5.4	2.0442 434	2.0442 434	0.0000																																												
2.75	8915 44	8951 43	9965	5.5	2.0876 434	2.0876 434	0.0000																																												
2.76	8959 44	8994 43	9965	5.6	2.1310 434	2.1310 434	0.0000																																												
2.77	9003 44	9037 43	9966	5.7	2.1744 434	2.1745 434	0.0000																																												
2.78	9046 44	9080 43	9967	5.8	2.2179 434	2.2179 434	0.0000																																												
2.79	9090 44	9123 43	9967	5.9	2.2613 434	2.2613 434	0.0000																																												
2.80	9134 44	9166 43	9968	6.0	2.3047 4343	2.3047 4343	0.0000																																												
2.81	9178 44	9209 43	9969	7.0	2.7390 4343	2.7390 4343	0.0000																																												
2.82	9221 44	9252 43	9969	8.0	3.1733 4343	3.1733 4343	0.0000																																												
2.83	9265 44	9295 43	9970	9.0	3.6076 4343	3.6076 4343	0.0000																																												
2.84	9309 44	9338 43	9970	10.0	4.0419 4343	4.0419 4343	0.0000																																												
2.85	9353 44	9382 43	9971	For higher values: $\log \text{Sh } x = \log \text{Ch } x = x\mu - 0.301030$; $\text{Sh}^{-1} u = \text{Ch}^{-1} u = (\log u + 0.2010) \mu^{-1}$.																																															
2.86	9396 44	9425 43	9972	<table><tr><th>n</th><th>$n\mu$</th><th>$n\mu^{-1}$</th><th>n</th></tr><tr><td>1</td><td>0.434294</td><td>2.302585</td><td>1</td></tr><tr><td>2</td><td>0.868589</td><td>4.605170</td><td>2</td></tr><tr><td>3</td><td>1.302883</td><td>6.907755</td><td>3</td></tr><tr><td>4</td><td>1.737178</td><td>9.210340</td><td>4</td></tr><tr><td>5</td><td>2.171472</td><td>11.512925</td><td>5</td></tr><tr><td>6</td><td>2.605767</td><td>13.815511</td><td>6</td></tr><tr><td>7</td><td>3.040061</td><td>16.118096</td><td>7</td></tr><tr><td>8</td><td>3.474356</td><td>18.420681</td><td>8</td></tr><tr><td>9</td><td>3.908650</td><td>20.723266</td><td>9</td></tr><tr><td>10</td><td>4.342945</td><td>23.025851</td><td>10</td></tr></table>				n	$n\mu$	$n\mu^{-1}$	n	1	0.434294	2.302585	1	2	0.868589	4.605170	2	3	1.302883	6.907755	3	4	1.737178	9.210340	4	5	2.171472	11.512925	5	6	2.605767	13.815511	6	7	3.040061	16.118096	7	8	3.474356	18.420681	8	9	3.908650	20.723266	9	10	4.342945	23.025851	10
n	$n\mu$	$n\mu^{-1}$	n																																																
1	0.434294	2.302585	1																																																
2	0.868589	4.605170	2																																																
3	1.302883	6.907755	3																																																
4	1.737178	9.210340	4																																																
5	2.171472	11.512925	5																																																
6	2.605767	13.815511	6																																																
7	3.040061	16.118096	7																																																
8	3.474356	18.420681	8																																																
9	3.908650	20.723266	9																																																
10	4.342945	23.025851	10																																																
2.87	9440 44	9468 43	9972																																																
2.88	9484 44	9511 43	9973																																																
2.89	9527 44	9554 43	9973																																																
2.90	9571 44	9597 43	9974																																																
2.91	9615 44	9641 43	9974																																																
2.92	9658 44	9684 43	9975																																																
2.93	9702 44	9727 43	9975																																																
2.94	9746 44	9770 43	9976																																																
2.95	9789 44	9813 43	9976																																																
2.96	9833 44	9856 43	9977																																																
2.97	9877 44	9900 43	9977																																																
2.98	9920 44	9943 43	9978																																																
2.99	9964 44	9986 43	9978																																																
3.00	1.0008 44	1.0029 43	9978																																																

Natural Sines and Cosines.

ϕ	$\sin \phi$	$\cos \phi$		ϕ	$\sin \phi$	$\cos \phi$		ϕ	$\sin \phi$	$\cos \phi$	
0°	.000000	1.0000	90°	7°				15°	.2588	.9659	75°
10'	.002909	1.0000	50'	30'	.1305	.9914	30'	10'	.2618	.9652	50'
20'	.005818	1.0000	40'	40'	.1334	.9911	20'	20'	.2644	.9644	40'
30'	.008727	1.0000	30'	50'	.1363	.9907	10'	30'	.2672	.9636	30'
40'	.011635	0.9999	20'	8°	.1392	.9903	82°	40'	.2700	.9628	20'
50'	.014544	0.9999	10'	10'	.1421	.9899	50'	50'	.2728	.9621	10'
1°	.017452	0.9998	89°	20'	.1449	.9894	40'	16°	.2756	.9613	74°
10'	.02036	0.9998	50'	30'	.1478	.9890	30'	10'	.2784	.9605	50'
20'	.02327	0.9997	40'	40'	.1507	.9886	20'	20'	.2812	.9596	40'
30'	.02618	0.9997	30'	50'	.1536	.9881	10'	30'	.2840	.9588	30'
40'	.02908	0.9996	20'	9°	.1564	.9877	81°	40'	.2868	.9580	20'
50'	.03199	0.9995	10'	10'	.1593	.9872	50'	50'	.2896	.9572	10'
2°	.03490	0.9994	88°	20'	.1622	.9868	40'	17°	.2924	.9563	73°
10'	.03781	0.9993	50'	30'	.1650	.9863	30'	10'	.2952	.9555	50'
20'	.04071	0.9992	40'	40'	.1679	.9858	20'	20'	.2979	.9546	40'
30'	.04362	0.9990	30'	50'	.1708	.9853	10'	30'	.3007	.9537	30'
40'	.04653	0.9989	20'	10°	.1736	.9848	80°	40'	.3035	.9528	20'
50'	.04943	0.9988	10'	10'	.1765	.9843	50'	50'	.3062	.9520	10'
3°	.05234	0.9986	87°	20'	.1794	.9838	40'	18°	.3090	.9511	72°
10'	.05524	0.9985	50'	30'	.1822	.9833	30'	10'	.3118	.9502	50'
20'	.05814	0.9983	40'	40'	.1851	.9827	20'	20'	.3145	.9492	40'
30'	.06105	0.9981	30'	50'	.1880	.9822	10'	30'	.3173	.9483	30'
40'	.06395	0.9980	20'	11°	.1908	.9816	79°	40'	.3201	.9474	20'
50'	.06685	0.9978	10'	10'	.1937	.9811	50'	50'	.3228	.9465	10'
4°	.06976	0.9976	86°	20'	.1965	.9805	40'	19°	.3256	.9455	71°
10'	.07266	0.9974	50'	30'	.1994	.9799	30'	10'	.3283	.9446	50'
20'	.07556	0.9971	40'	40'	.2022	.9793	20'	20'	.3311	.9436	40'
30'	.07846	0.9969	30'	50'	.2051	.9787	10'	30'	.3338	.9426	30'
40'	.08136	0.9967	20'	12°	.2079	.9781	78°	40'	.3365	.9417	20'
50'	.08426	0.9964	10'	10'	.2108	.9775	50'	50'	.3393	.9407	10'
5°	.08716	0.9962	85°	20'	.2136	.9769	40'	20°	.3420	.9397	70°
10'	.09005	0.9959	50'	30'	.2164	.9763	30'	10'	.3448	.9387	50'
20'	.09295	0.9957	40'	40'	.2193	.9757	20'	20'	.3475	.9377	40'
30'	.09585	0.9954	30'	50'	.2221	.9750	10'	30'	.3502	.9367	30'
40'	.09874	0.9951	20'	13°	.2250	.9744	77°	40'	.3529	.9356	20'
50'	.10164	0.9948	10'	10'	.2278	.9737	50'	50'	.3557	.9346	10'
6°	.10453	0.9945	84°	20'	.2306	.9730	40'	21°	.3584	.9336	69°
10'	.1074	0.9942	50'	30'	.2334	.9724	30'	10'	.3611	.9325	50'
20'	.1103	0.9939	40'	40'	.2363	.9717	20'	20'	.3638	.9315	40'
30'	.1132	0.9936	30'	50'	.2391	.9710	10'	30'	.3665	.9304	30'
40'	.1161	0.9932	20'	14°	.2419	.9703	76°	40'	.3692	.9293	20'
50'	.1190	0.9929	10'	10'	.2447	.9696	50'	50'	.3719	.9283	10'
7°	.1219	0.9925	83°	20'	.2476	.9689	40'	22°	.3746	.9272	68°
10'	.1248	0.9922	50'	30'	.2504	.9681	30'	10'	.3773	.9261	50'
20'	.1276	0.9918	40'	40'	.2532	.9674	20'	20'	.3800	.9250	40'
30'	.1305	0.9914	30'	50'	.2560	.9667	10'	30'	.3827	.9239	30'
			82°	15°	.2588	.9659	75°				67°
	$\cos \theta$	$\sin \theta$	θ		$\cos \theta$	$\sin \theta$	θ		$\cos \theta$	$\sin \theta$	θ

Natural Sines and Cosines.

ϕ	$\sin \phi$	$\cos \phi$		ϕ	$\sin \phi$	$\cos \phi$		ϕ	$\sin \phi$	$\cos \phi$	
22°				30°	.5000	.8660		60°	37°		
30'	.3827	.9239	30'	10'	.5025	.8646	50'	30'	.6088	.7934	30'
40'	.3854	.9228	20'	20'	.5050	.8631	40'	40'	.6111	.7916	20'
50'	.3881	.9216	10'	30'	.5075	.8616	30'	50'	.6134	.7898	10'
23°	.3907	.9205	67°	40'	.5100	.8601	20'	38°	.6157	.7880	52°
10'	.3934	.9194	50'	50'	.5125	.8587	10'	10'	.6180	.7862	50'
20'	.3961	.9182	40'	31°	.5150	.8572	59°	20'	.6202	.7844	40'
30'	.3987	.9171	30'	10'	.5175	.8557	50'	30'	.6225	.7826	30'
40'	.4014	.9159	20'	20'	.5200	.8542	40'	40'	.6248	.7808	20'
50'	.4041	.9147	10'	30'	.5225	.8526	30'	50'	.6271	.7790	10'
24°	.4067	.9135	66°	40'	.5250	.8511	20'	39°	.6293	.7771	51°
10'	.4094	.9124	50'	50'	.5275	.8496	10'	10'	.6316	.7753	50'
20'	.4120	.9112	40'	32°	.5299	.8480	58°	20'	.6338	.7735	40'
30'	.4147	.9100	30'	10'	.5324	.8465	50'	30'	.6361	.7716	30'
40'	.4173	.9088	20'	20'	.5348	.8450	40'	40'	.6383	.7698	20'
50'	.4200	.9075	10'	30'	.5373	.8434	30'	50'	.6406	.7679	10'
25°	.4226	.9063	65°	40'	.5398	.8418	20'	40°	.6428	.7660	50°
10'	.4253	.9051	50'	50'	.5422	.8403	10'	10'	.6450	.7642	50'
20'	.4279	.9038	40'	33°	.5446	.8387	57°	20'	.6472	.7623	40'
30'	.4305	.9026	30'	10'	.5471	.8371	50'	30'	.6494	.7604	30'
40'	.4331	.9013	20'	20'	.5495	.8355	40'	40'	.6517	.7585	20'
50'	.4358	.9001	10'	30'	.5519	.8339	30'	50'	.6539	.7566	10'
26°	.4384	.8988	64°	40'	.5544	.8323	20'	41°	.6561	.7547	49°
10'	.4410	.8975	50'	50'	.5568	.8307	10'	10'	.6583	.7528	50'
20'	.4436	.8962	40'	34°	.5592	.8290	56°	20'	.6604	.7509	40'
30'	.4462	.8949	30'	10'	.5616	.8274	50'	30'	.6626	.7490	30'
40'	.4488	.8936	20'	20'	.5640	.8258	40'	40'	.6648	.7470	20'
50'	.4514	.8923	10'	30'	.5664	.8241	30'	50'	.6670	.7451	10'
27°	.4540	.8910	63°	40'	.5688	.8225	20'	42°	.6691	.7431	48°
10'	.4566	.8897	50'	50'	.5712	.8208	10'	10'	.6713	.7412	50'
20'	.4592	.8884	40'	35°	.5736	.8192	55°	20'	.6734	.7392	40'
30'	.4617	.8870	30'	10'	.5760	.8175	50'	30'	.6756	.7373	30'
40'	.4643	.8857	20'	20'	.5783	.8158	40'	40'	.6777	.7353	20'
50'	.4669	.8843	10'	30'	.5807	.8141	30'	50'	.6799	.7333	10'
28°	.4695	.8829	62°	40'	.5831	.8124	20'	43°	.6820	.7314	47°
10'	.4720	.8816	50'	50'	.5854	.8107	10'	10'	.6841	.7294	50'
20'	.4746	.8802	40'	36°	.5878	.8090	54°	20'	.6862	.7274	40'
30'	.4772	.8788	30'	10'	.5901	.8073	50'	30'	.6884	.7254	30'
40'	.4797	.8774	20'	20'	.5925	.8056	40'	40'	.6905	.7234	20'
50'	.4823	.8760	10'	30'	.5948	.8039	30'	50'	.6926	.7214	10'
29°	.4848	.8746	61°	40'	.5972	.8021	20'	44°	.6947	.7193	46°
10'	.4874	.8732	50'	50'	.5995	.8004	10'	10'	.6967	.7173	50'
20'	.4899	.8718	40'	37°	.6018	.7986	53°	20'	.6988	.7153	40'
30'	.4924	.8704	30'	10'	.6041	.7969	50'	30'	.7009	.7133	30'
40'	.4950	.8689	20'	20'	.6065	.7951	40'	40'	.7030	.7112	20'
50'	.4975	.8675	10'	30'	.6088	.7934	30'	50'	.7050	.7092	10'
30°	.5000	.8660	60°				52°	45°	.7071	.7071	45°
	$\cos \theta$	$\sin \theta$	θ		$\cos \theta$	$\sin \theta$	θ		$\cos \theta$	$\sin \theta$	θ

Natural Tangents and Cotangents.

ϕ	$\tan \phi$	$\cot \phi$		ϕ	$\tan \phi$	$\cot \phi$		ϕ	$\tan \phi$	$\cot \phi$	
0°	.000000		90°	7°				15°	.2679	3.732	75°
10'	.002909		50'	30'	.1317	7.60	30'	10'	.2711	3.689	50'
20'	.005818		40'	40'	.1346	7.43	20'	20'	.2742	3.647	40'
30'	.008727		30'	50'	.1376	7.27	10'	30'	.2773	3.606	30'
40'	.011636		20'	8°	.1405	7.12	82°	40'	.2805	3.566	20'
50'	.014545		10'	10'	.1435	6.97	50'	50'	.2836	3.526	10'
1°	.017455	57.	89°	20'	.1465	6.83		16°	.2867	3.487	74°
10'	.02036	49.	50'	30'	.1495	6.69	30'	10'	.2899	3.450	50'
20'	.02328	43.	40'	40'	.1524	6.56	20'	20'	.2931	3.412	40'
30'	.02619	38.	30'	50'	.1554	6.43	10'	30'	.2962	3.376	30'
40'	.02910	34.	20'	9°	.1584	6.31	81°	40'	.2994	3.340	20'
50'	.03201	31.	10'	10'	.1614	6.197	50'	50'	.3026	3.305	10'
2°	.03492	28.6	88°	20'	.1644	6.084		17°	.3057	3.271	73°
10'	.03783	26.4	50'	30'	.1673	5.976	30'	10'	.3089	3.237	50'
20'	.04075	24.5	40'	40'	.1703	5.871	20'	20'	.3121	3.204	40'
30'	.04366	22.9	30'	50'	.1733	5.769	10'	30'	.3153	3.172	30'
40'	.04658	21.5	20'	10°	.1763	5.671	80°	40'	.3185	3.140	20'
50'	.04949	20.2	10'	10'	.1793	5.576	50'	50'	.3217	3.108	10'
3°	.05241	19.1	87°	20'	.1823	5.485		18°	.3249	3.078	72°
10'	.05533	18.1	50'	30'	.1853	5.396	30'	10'	.3281	3.047	50'
20'	.05824	17.2	40'	40'	.1883	5.309	20'	20'	.3314	3.018	40'
30'	.06116	16.3	30'	50'	.1914	5.226	10'	30'	.3346	2.989	30'
40'	.06408	15.6	20'	11°	.1944	5.145	79°	40'	.3378	2.960	20'
50'	.06700	14.9	10'	10'	.1974	5.066	50'	50'	.3411	2.932	10'
4°	.06993	14.3	86°	20'	.2004	4.989		19°	.3443	2.904	71°
10'	.07285	13.73	50'	30'	.2035	4.915	30'	10'	.3476	2.877	50'
20'	.07578	13.20	40'	40'	.2065	4.843	20'	20'	.3508	2.850	40'
30'	.07870	12.71	30'	50'	.2095	4.773	10'	30'	.3541	2.824	30'
40'	.08163	12.25	20'	12°	.2126	4.705	78°	40'	.3574	2.798	20'
50'	.08456	11.83	10'	10'	.2156	4.638	50'	50'	.3607	2.773	10'
5°	.08749	11.43	85°	20'	.2186	4.574		20°	.3640	2.747	70°
10'	.09042	11.06	50'	30'	.2217	4.511	30'	10'	.3673	2.723	50'
20'	.09335	10.71	40'	40'	.2247	4.449	20'	20'	.3706	2.699	40'
30'	.09629	10.39	30'	50'	.2278	4.390	10'	30'	.3739	2.675	30'
40'	.09923	10.08	20'	13°	.2309	4.331	77°	40'	.3772	2.651	20'
50'	.10216	9.79	10'	10'	.2339	4.275	50'	50'	.3805	2.628	10'
6°	.10510	9.51	84°	20'	.2370	4.219		21°	.3839	2.605	69°
10'	.1080	9.26	50'	30'	.2401	4.165	30'	10'	.3872	2.583	50'
20'	.1110	9.01	40'	40'	.2432	4.113	20'	20'	.3906	2.560	40'
30'	.1139	8.78	30'	50'	.2462	4.061	10'	30'	.3939	2.539	30'
40'	.1169	8.56	20'	14°	.2493	4.011	76°	40'	.3973	2.517	20'
50'	.1198	8.34	10'	10'	.2524	3.962	50'	50'	.4006	2.496	10'
7°	.1228	8.14	83°	20'	.2555	3.914		22°	.4040	2.475	68°
10'	.1257	7.95	50'	30'	.2586	3.867	30'	10'	.4074	2.455	50'
20'	.1287	7.77	40'	40'	.2617	3.821	20'	20'	.4108	2.434	40'
30'	.1317	7.60	30'	50'	.2648	3.776	10'	30'	.4142	2.414	30'
			82°	15°	.2679	3.732	75°				67°
$\cot \theta$ $\tan \theta$ θ				$\cot \theta$ $\tan \theta$ θ				$\cot \theta$ $\tan \theta$ θ			

Natural Tangents and Cotangents.

ϕ	$\tan \phi$	$\cot \phi$		ϕ	$\tan \phi$	$\cot \phi$		ϕ	$\tan \phi$	$\cot \phi$	
22°				30°	.5774	1.732		60°	37°		
30'	.4142	2.414	30'	10'	.5812	1.720	50'	30'	.7673	1.303	30'
40'	.4176	2.394	20'	20'	.5851	1.709	40'	40'	.7720	1.295	20'
50'	.4210	2.375	10'	30'	.5890	1.698	30'	50'	.7766	1.288	10'
23°	.4245	2.356	67°	40'	.5930	1.686	20'	38°	.7813	1.280	52°
10'	.4279	2.337	50'	50'	.5969	1.675	10'	10'	.7860	1.272	50'
20'	.4314	2.318	40'	31°	.6009	1.664	59°	20'	.7907	1.265	40'
30'	.4348	2.300	30'	10'	.6048	1.653	50'	30'	.7954	1.257	30'
40'	.4383	2.282	20'	20'	.6088	1.643	40'	40'	.8002	1.250	20'
50'	.4417	2.264	10'	30'	.6128	1.632	30'	50'	.8050	1.242	10'
24°	.4452	2.246	66°	40'	.6168	1.621	20'	39°	.8098	1.235	51°
10'	.4487	2.229	50'	50'	.6208	1.611	10'	10'	.8146	1.228	50'
20'	.4522	2.211	40'	32°	.6249	1.600	58°	20'	.8195	1.220	40'
30'	.4557	2.194	30'	10'	.6289	1.590	50'	30'	.8243	1.213	30'
40'	.4592	2.177	20'	20'	.6330	1.580	40'	40'	.8292	1.206	20'
50'	.4628	2.161	10'	30'	.6371	1.570	30'	50'	.8342	1.199	10'
25°	.4663	2.145	65°	40'	.6412	1.560	20'	40°	.8391	1.192	50°
10'	.4699	2.128	50'	50'	.6453	1.550	10'	10'	.8441	1.185	50'
20'	.4734	2.112	40'	33°	.6494	1.540	57°	20'	.8491	1.178	40'
30'	.4770	2.097	30'	10'	.6536	1.530	50'	30'	.8541	1.171	30'
40'	.4806	2.081	20'	20'	.6577	1.520	40'	40'	.8591	1.164	20'
50'	.4841	2.066	10'	30'	.6619	1.511	30'	50'	.8642	1.157	10'
26°	.4877	2.050	64°	40'	.6661	1.501	20'	41°	.8693	1.150	49°
10'	.4913	2.035	50'	50'	.6703	1.492	10'	10'	.8744	1.144	50'
20'	.4950	2.020	40'	34°	.6745	1.483	56°	20'	.8796	1.137	40'
30'	.4986	2.006	30'	10'	.6787	1.473	50'	30'	.8847	1.130	30'
40'	.5022	1.991	20'	20'	.6830	1.464	40'	40'	.8899	1.124	20'
50'	.5059	1.977	10'	30'	.6873	1.455	30'	50'	.8952	1.117	10'
27°	.5095	1.963	63°	40'	.6916	1.446	20'	42°	.9004	1.111	48°
10'	.5132	1.949	50'	50'	.6959	1.437	10'	10'	.9057	1.104	50'
20'	.5169	1.935	40'	35°	.7002	1.428	55°	20'	.9110	1.098	40'
30'	.5206	1.921	30'	10'	.7046	1.419	50'	30'	.9163	1.091	30'
40'	.5243	1.907	20'	20'	.7089	1.411	40'	40'	.9217	1.085	20'
50'	.5280	1.894	10'	30'	.7133	1.402	30'	50'	.9271	1.079	10'
28°	.5317	1.881	62°	40'	.7177	1.393	20'	43°	.9325	1.072	47°
10'	.5354	1.868	50'	50'	.7221	1.385	10'	10'	.9380	1.066	50'
20'	.5392	1.855	40'	36°	.7265	1.376	54°	20'	.9435	1.060	40'
30'	.5430	1.842	30'	10'	.7310	1.368	50'	30'	.9490	1.054	30'
40'	.5467	1.829	20'	20'	.7355	1.360	40'	40'	.9545	1.048	20'
50'	.5505	1.816	10'	30'	.7400	1.351	30'	50'	.9601	1.042	10'
29°	.5543	1.804	61°	40'	.7445	1.343	20'	44°	.9657	1.036	46°
10'	.5581	1.792	50'	50'	.7490	1.335	10'	10'	.9713	1.030	50'
20'	.5619	1.780	40'	37°	.7536	1.327	53°	20'	.9770	1.024	40'
30'	.5658	1.767	30'	10'	.7581	1.319	50'	30'	.9827	1.018	30'
40'	.5696	1.756	20'	20'	.7627	1.311	40'	40'	.9884	1.012	20'
50'	.5735	1.744	10'	30'	.7673	1.303	30'	50'	.9942	1.006	10'
30°	.5774	1.732	60°				52°	45°	1.0000	1.000	45°
$\cot \theta \quad \tan \theta \quad \theta$				$\cot \theta \quad \tan \theta \quad \theta$				$\cot \theta \quad \tan \theta \quad \theta$			

Natural Secants and Cosecants.

ϕ	sec ϕ	csc ϕ		ϕ	sec ϕ	csc ϕ		ϕ	sec ϕ	csc ϕ	
0°	1.0000		90°	7°				15°	1.035	3.864	75°
10'	1.0000		50'	30'	1.009	7.86	30'	10'	1.036	3.822	60'
20'	1.0000		40'	40'	1.009	7.50	20'	20'	1.037	3.782	40'
30'	1.0000		30'	50'	1.009	7.34	10'	30'	1.038	3.742	30'
40'	1.0001		20'	8°	1.010	7.19	82°	40'	1.039	3.703	20'
50'	1.0001		10'	10'	1.010	7.04	50'	50'	1.039	3.665	10'
1°	1.0002	57.	89°	20'	1.011	6.90	30'	16°	1.040	3.628	74°
10'	1.0002	49.	50'	30'	1.011	6.77	40'	10'	1.041	3.592	60'
20'	1.0003	43.	40'	40'	1.012	6.64	20'	20'	1.042	3.556	40'
30'	1.0003	38.	30'	50'	1.012	6.51	10'	30'	1.043	3.521	30'
40'	1.0004	34.	20'	9°	1.012	6.39	81°	40'	1.044	3.487	20'
50'	1.0005	31.	10'	10'	1.013	6.277	50'	50'	1.045	3.453	10'
2°	1.0006	28.7	88°	20'	1.013	6.166	30'	17°	1.046	3.420	73°
10'	1.0007	26.5	50'	30'	1.014	6.069	40'	10'	1.047	3.388	60'
20'	1.0008	24.6	40'	40'	1.014	5.965	20'	20'	1.048	3.356	40'
30'	1.0010	22.9	30'	50'	1.015	5.865	10'	30'	1.049	3.326	30'
40'	1.0011	21.5	20'	10°	1.015	5.769	80°	40'	1.049	3.295	20'
50'	1.0012	20.2	10'	10'	1.016	5.665	50'	50'	1.050	3.265	10'
3°	1.0014	19.1	87°	20'	1.016	5.575	30'	18°	1.051	3.236	72°
10'	1.0015	18.1	50'	30'	1.017	5.487	40'	10'	1.052	3.207	60'
20'	1.0017	17.2	40'	40'	1.018	5.403	20'	20'	1.053	3.179	40'
30'	1.0019	16.4	30'	50'	1.018	5.320	10'	30'	1.054	3.152	30'
40'	1.0021	15.6	20'	11°	1.019	5.241	79°	40'	1.056	3.124	20'
50'	1.0022	15.0	10'	10'	1.019	5.164	50'	50'	1.057	3.098	10'
4°	1.0024	14.3	86°	20'	1.020	5.089	30'	19°	1.058	3.072	71°
10'	1.0026	13.76	50'	30'	1.020	5.016	40'	10'	1.059	3.046	60'
20'	1.0029	13.23	40'	40'	1.021	4.945	20'	20'	1.060	3.021	40'
30'	1.0031	12.75	30'	50'	1.022	4.876	10'	30'	1.061	2.996	30'
40'	1.0033	12.29	20'	12°	1.022	4.810	78°	40'	1.062	2.971	20'
50'	1.0036	11.87	10'	10'	1.023	4.745	50'	50'	1.063	2.947	10'
5°	1.0038	11.47	85°	20'	1.024	4.682	30'	20°	1.064	2.924	70°
10'	1.0041	11.10	50'	30'	1.024	4.620	40'	10'	1.065	2.901	60'
20'	1.0043	10.76	40'	40'	1.025	4.560	20'	20'	1.066	2.878	40'
30'	1.0046	10.43	30'	50'	1.026	4.502	10'	30'	1.068	2.855	30'
40'	1.0049	10.13	20'	13°	1.026	4.445	77°	40'	1.069	2.833	20'
50'	1.0052	9.84	10'	10'	1.027	4.390	50'	50'	1.070	2.812	10'
6°	1.0055	9.57	84°	20'	1.028	4.336	30'	21°	1.071	2.790	69°
10'	1.0058	9.31	50'	30'	1.028	4.284	40'	10'	1.072	2.769	60'
20'	1.0061	9.07	40'	40'	1.029	4.232	20'	20'	1.074	2.749	40'
30'	1.0065	8.83	30'	50'	1.030	4.182	10'	30'	1.075	2.729	30'
40'	1.0068	8.61	20'	14°	1.031	4.134	76°	40'	1.076	2.709	20'
50'	1.0072	8.40	10'	10'	1.031	4.086	50'	50'	1.077	2.689	10'
7°	1.0075	8.21	83°	20'	1.032	4.039	30'	22°	1.079	2.669	68°
10'	1.0079	8.02	50'	30'	1.033	3.994	40'	10'	1.080	2.650	60'
20'	1.0082	7.83	40'	40'	1.034	3.950	20'	20'	1.081	2.632	40'
30'	1.0086	7.66	30'	50'	1.034	3.906	10'	30'	1.082	2.613	30'
			82°	15°	1.035	3.864	75°				67°
csc θ sec θ θ				csc θ sec θ θ				csc θ sec θ θ			

Natural Secants and Cosecants.

ϕ	sec ϕ	csc ϕ		ϕ	sec ϕ	csc ϕ		ϕ	sec ϕ	csc ϕ	
22°				30°	1.155	2.000		60°			
30'	1.082	2.613	30'	10'	1.157	1.990	50'	37°	1.260	1.643	30'
40'	1.084	2.595	20'	20'	1.159	1.980	40'	40'	1.263	1.636	20'
50'	1.085	2.577	10'	30'	1.161	1.970	30'	50'	1.266	1.630	10'
23°	1.086	2.559	67°	40'	1.163	1.961	20'	38°	1.269	1.624	52°
10'	1.088	2.542	50'	50'	1.165	1.951	10'	10'	1.272	1.618	50'
20'	1.089	2.525	40'	31°	1.167	1.942	59°	20'	1.275	1.612	40'
30'	1.090	2.508	30'	10'	1.169	1.932	50'	30'	1.278	1.606	30'
40'	1.092	2.491	20'	20'	1.171	1.923	40'	40'	1.281	1.601	20'
50'	1.093	2.475	10'	30'	1.173	1.914	30'	50'	1.284	1.595	10'
24°	1.095	2.459	66°	40'	1.175	1.905	20'	39°	1.287	1.589	51°
10'	1.096	2.443	50'	50'	1.177	1.896	10'	10'	1.290	1.583	50'
20'	1.097	2.427	40'	32°	1.179	1.887	58°	20'	1.293	1.578	40'
30'	1.099	2.411	30'	10'	1.181	1.878	50'	30'	1.296	1.572	30'
40'	1.100	2.396	20'	20'	1.184	1.870	40'	40'	1.299	1.567	20'
50'	1.102	2.381	10'	30'	1.186	1.861	30'	50'	1.302	1.561	10'
25°	1.103	2.366	65°	40'	1.188	1.853	20'	40°	1.305	1.556	50°
10'	1.105	2.352	50'	50'	1.190	1.844	10'	10'	1.309	1.550	50'
20'	1.106	2.337	40'	33°	1.192	1.836	57°	20'	1.312	1.545	40'
30'	1.108	2.323	30'	10'	1.195	1.828	50'	30'	1.315	1.540	30'
40'	1.109	2.309	20'	20'	1.197	1.820	40'	40'	1.318	1.535	20'
50'	1.111	2.296	10'	30'	1.199	1.812	30'	50'	1.322	1.529	10'
26°	1.113	2.281	64°	40'	1.202	1.804	20'	41°	1.325	1.524	49°
10'	1.114	2.268	50'	50'	1.204	1.796	10'	10'	1.328	1.519	50'
20'	1.116	2.254	40'	34°	1.206	1.788	56°	20'	1.332	1.514	40'
30'	1.117	2.241	30'	10'	1.209	1.781	50'	30'	1.335	1.509	30'
40'	1.119	2.228	20'	20'	1.211	1.773	40'	40'	1.339	1.504	20'
50'	1.121	2.215	10'	30'	1.213	1.766	30'	50'	1.342	1.499	10'
27°	1.122	2.203	63°	40'	1.216	1.758	20'	42°	1.346	1.494	48°
10'	1.124	2.190	50'	50'	1.218	1.751	10'	10'	1.349	1.490	50'
20'	1.126	2.178	40'	35°	1.221	1.743	55°	20'	1.353	1.485	40'
30'	1.127	2.166	30'	10'	1.223	1.736	50'	30'	1.356	1.480	30'
40'	1.129	2.154	20'	20'	1.226	1.729	40'	40'	1.360	1.476	20'
50'	1.131	2.142	10'	30'	1.228	1.722	30'	50'	1.364	1.471	10'
28°	1.133	2.130	62°	40'	1.231	1.715	20'	43°	1.367	1.466	47°
10'	1.134	2.118	50'	50'	1.233	1.708	10'	10'	1.371	1.462	50'
20'	1.136	2.107	40'	36°	1.236	1.701	54°	20'	1.375	1.457	40'
30'	1.138	2.096	30'	10'	1.239	1.695	50'	30'	1.379	1.453	30'
40'	1.140	2.085	20'	20'	1.241	1.688	40'	40'	1.382	1.448	20'
50'	1.142	2.074	10'	30'	1.244	1.681	30'	50'	1.386	1.444	10'
29°	1.143	2.063	61°	40'	1.247	1.675	20'	44°	1.390	1.440	46°
10'	1.145	2.052	50'	50'	1.249	1.668	10'	10'	1.394	1.435	50'
20'	1.147	2.041	40'	37°	1.252	1.662	53°	20'	1.398	1.431	40'
30'	1.149	2.031	30'	10'	1.255	1.655	50'	30'	1.402	1.427	30'
40'	1.151	2.020	20'	20'	1.258	1.649	40'	40'	1.406	1.423	20'
50'	1.153	2.010	10'	30'	1.260	1.643	30'	50'	1.410	1.418	10'
30°	1.155	2.000	60°				52°	45°	1.414	1.414	45°
csc θ sec θ θ				csc θ sec θ θ				csc θ sec θ θ			



EXPLANATION OF THE TABLES.

§ 1. TABLES IN GENERAL.

a. One quantity is said to be a **function** of another, when the former quantity is regarded as determined by the latter, according to some rule or formula. E. g. x^2 , \sqrt{x} , $\log x$, $\sin x$, $\log \sin x$, are all called functions of x . A **mathematical table** is an orderly arrangement of the values of some function for certain selected values of the quantity by which it is regarded as determined. The successive values of the latter quantity are assumed arbitrarily, and generally at equal intervals; and this quantity is called the **argument** of the table. Some functions require several independent quantities for their determination; and the corresponding tables are tables of several arguments. Thus, a multiplication-table is a table of *two* arguments; namely, the two factors.

b. A table may be used in two ways: **directly** and **inversely**. The direct use of the table consists in finding the value of the function for an assumed value of the argument; the inverse use, in finding the value of the argument for an assumed value of the function.

c. Before beginning to use any table, the student should give attentive consideration to its arrangement, and to the best mode of employing it with accuracy and ease. Every feature of it should be carefully examined, and the explanations which are attached to it should be fully mastered. The time thus spent will be time gained, contributing not only to power in computation, but also, very materially, to the thorough practical knowledge of the nature of the tabulated functions.

§ 2. INTERPOLATION.

a. **Interpolation** consists in finding the value of one of the two quantities, argument and function, for an assumed value of the other quantity, lying between two successive tabulated values. Most mathematical tables are so constructed as to admit of interpolation by the principle that *corresponding non-tabulated values of the function and argument lie between corresponding tabulated values and divide the differences between them in the same ratio*. This is the principle of **proportional parts**. Let x_1 and x_2 be two successive tabulated values of the argument of a table, and u_1 and u_2 the correspond-

Explanation of the Tables.

ing values of the function. Then, $x_2 - x_1$ and $u_2 - u_1$ are called corresponding tabular differences. We shall denote these differences by Δx and Δu . If, now, x and u are corresponding values of the function and argument, of which one is known to lie between the two above-cited tabulated values of the same quantity, the principle of proportional parts is that if

$$\lambda = \frac{x - x_1}{\Delta x}, \quad \lambda' = \frac{x_2 - x}{\Delta x} = 1 - \lambda.$$

$$\mu = \frac{u - u_1}{\Delta u}, \quad \mu' = \frac{u_2 - u}{\Delta u} = 1 - \mu,$$

then (to the limit of accuracy belonging to the table)

$$\lambda = \mu, \quad \lambda' = \mu',$$

or,

$$u = u_1 + \lambda \Delta u = u_2 - \lambda' \Delta u,$$

$$x = x_1 + \mu \Delta x = x_2 - \mu' \Delta x.$$

Thus, the required value of the function or argument may be obtained by applying a **correction** to either of the two tabulated values between which the required value lies. In computing this correction, the *signs* of the differences employed must be carefully observed. If x_1 and x_2 are so chosen as to make Δx positive, Δu may be either positive or negative. In the former case, the function is said to be *increasing*; in the latter, *decreasing*.

b. Either of the two formulas given above for finding u may be employed, in interpolation, in the direct use of the table; either of the formulas for x may be employed in the inverse use of the table. In most tables, $\Delta x =$ one unit in the last numeral place of the tabulated values of x . Hence λ is composed of the figures which follow that numeral place in the given non-tabulated value of the argument, preceded by a decimal-point; while λ' is the complement of λ (that is, can be found by subtracting from 9 each figure of λ except the last, and subtracting that from 10). The correction for u is, therefore, found simply by multiplying the figures in question into Δu , and pointing off according to the case; x will be corrected by annexing to x_1 the figures of $\frac{u - u_1}{\Delta u}$, or the figures complementary to $\frac{u_2 - u}{\Delta u}$.

c. In some of the tables of this collection will be found, set against each value of the function, a number in small type, which shows what Δu would be if the function varied through a whole interval corresponding to Δx at the same rate at which it is changing when it passes through the value against which this number is set. This number may be called the *rate of difference*, or simply the *difference*, of u , and may be substituted for Δu in the formulas of interpolation. But, in that case, we ought to work from the NEAREST tabulated value of x or u ; that is, from x_1 or u_1 when λ or $\mu < 0.5$, and from x_2 or u_2 when λ' or $\mu' < 0.5$. (See examples in the explanation of the table of *Logarithms of Circular Functions*.)

d. An interpolated value of the function should not be carried out beyond the last numeral place of the tabulated value from which it is computed; so that, in finding $\lambda \Delta u$ or $\lambda' \Delta u$, we should reject the decimal part of the product, Δu being regarded as an integer. Owing to the combination of the figures rejected in the correction and those omitted in the tabulated value of the function, an interpolated value is liable to an error of ± 1 in the last figure.

Proportional Parts.

The number of figures annexed to the tabulated value of the argument, in *inverse interpolation*, should be less by one than the number of figures contained in Δu . It is sometimes, indeed, made equal to the latter number (and will always be, if Δu consists of only one figure); but, in that case, the *last figure* must be regarded as *uncertain*. When the given value of the function is the result of computation, of course this uncertainty may extend back to earlier figures.

e. In taking the correction of either the function or the argument only to a certain number of figures, we must observe the following rule, which is a universal rule of computation:—

Whenever figures are neglected at the end of a number, if the figures neglected amount to more than half a unit in the place of the last figure retained, the last figure retained must be increased by 1. E. g. $27.528 = 27.53$ to the nearest hundredth $= 27.5$ to the nearest tenth $= 28$ to the nearest unit $= 80$ to the nearest ten.

f. The various rules of interpolation will be found to be fully exemplified below, in the explanations of the tables of *Logarithms* and *Logarithms of Circular Functions*.

g. In interpolating in some tables (e. g. in VLACQ's great ten-place table), we must have regard to **second differences**, or differences between differences. In this case, we add to the above formulas for u the term

$$-\frac{1}{2} \lambda \lambda' \Delta^2 u,$$

where $\Delta^2 u$ denotes the second difference of u , taken positively when Δu is increasing. The greatest value of this term is *one eighth* of $\Delta^2 u$, so that it is insignificant when $\Delta^2 u < 4$. In the present tables this term may always be neglected; although it is useful as measuring the extent of error, and may occasionally guide the judgment of the computer when the fractional part of the correction is equal, or nearly equal, to 0.5. But where such nicety of work seems to be called for, it is best to use a table of a larger number of places.

§ 3. PROPORTIONAL PARTS.

a. The table of **Proportional Parts** (folded page) may be used in connexion with any other table, as an aid in *interpolation*. It contains the product of every integer from 1 to 100 by every *tenth* from 0.1 to 0.9. If the multiplier consists of one figure in any other numeral place, it is only necessary to change the position of the decimal-point in the product. To multiply a number of two figures by any decimal whatever, we must find the products which correspond to the successive figures of the multiplier, and add them together. The decimal part of the result is generally to be discarded, and in that case the general rule given above (in § 2, e) must be observed. Thus, let it be required to find 0.619×37 . Looking in the column belonging to 37, we find

$$\begin{array}{rcl} 0.6 & \times 37 & = 22.2 \\ 0.01 & \times 37 & = 0.37 \\ 0.009 & \times 37 & = \underline{0.333} \\ \therefore 0.619 & \times 37 & = 23. \end{array}$$

Explanation of the Tables.

In like manner, we find

$$\begin{array}{lll} 0.27 \times 15 = 4, & 0.59 \times 73 = 43, & 0.78 \times 69 = 54, \\ 0.96 \times 84 = 81, & 0.36 \times 57 = 21, & 0.289 \times 51 = 15, \\ 0.483 \times 93 = 45, & 0.374 \times 82 = 31, & 0.053 \times 68 = 4. \end{array}$$

b. This table can also be used *inversely*. Thus, let it be required to find, to two decimal-places, what part 36 is of 79. Looking in the column of 79, we find

$$\begin{array}{r} 36 \\ 0.4 \times 79 = \frac{31.6}{4.4} \end{array}$$

$$0.06 \times 79 = 4.74 \text{ (the nearest product)}$$

$$\therefore \frac{36}{79} = 0.46.$$

In like manner, we find

$$\frac{29}{68} = 0.43, \quad \frac{72}{89} = 0.81, \quad \frac{31}{98} = 0.32, \quad \frac{26}{71} = 0.37, \quad \frac{45}{57} = 0.79, \quad \frac{11}{37} = 0.30.$$

A little practice will enable the student to use this table easily and rapidly.

§ 4. LOGARITHMS.

a. **Denary, or Briggsian,** logarithms, being those employed in actual computation, are always referred to, in this collection of tables, when the term *logarithm* is used without qualification. The **characteristic**, or integral part, of the denary logarithm of a number depends only on the position of the first significant figure of the number relatively to the units' place, and may be found by a well-known rule; the **mantissa**, or fractional part, depends only on the series of significant figures which compose the number, and is the only part of the logarithm for which it is necessary to employ a table. A table of logarithms is complete, to an assigned number of places, if it gives (explicitly or by interpolation) to that number of places the mantissa of the logarithm of every possible series of significant figures. Denary logarithms are, in general, incommensurable numbers, and cannot, therefore, be exactly expressed in figures. They are variously given, in different tables, to ten, seven, six, five, four, and three places of decimals. Four-place logarithms are sufficient for the ordinary purposes of engineering, navigation, the work of the physical and chemical laboratory, and many of the subordinate computations of astronomy; and, in most of these cases, are all that the accuracy of the data will justify us in using. Seven places are, however, needful for the more accurate kinds of astronomical and geodetic work.

b. If one number is the logarithm of another, the second number is called the **antilogarithm** of the first. This relation is denoted by the symbol \log^{-1} . Thus, if $u = \log x$, then $x = \log^{-1} u$. In an ordinary table of *logarithms*, the *argument* is the antilogarithm, which is tabulated to a *greater or less* number of figures, according to the number of places to

Logarithms.

which the logarithm is given, and the *function* is the mantissa of the logarithm, which we often speak of simply as the *logarithm*.

To find the logarithm of any number.

c. If the number consists of *three significant figures*, seek the first two significant figures in the first column of the table of **Logarithms** (pp. 2, 3), and the third at the top of the table. In the line and column thus determined will be found the mantissa of the required logarithm, printed without the decimal-point. Find the characteristic by the rule, and prefix it, with the decimal-point, to the mantissa. E. g., $\log 2870 = 3.4579$. If the given number has *less than three* significant figures, fill it out to three figures by annexing a zero or zeros. E. g., $\log 0.35 = \log 0.350 = 9.5441 - 10$, $\log 6 = \log 6.00 = 0.7782$. If the number has *more than three* significant figures, its logarithm must be found by one of the formulas of interpolation given above. The rule is: — *Find the logarithm of the first three significant figures of the given number and also that of the next following number of three figures (1000 following 999); then apply to EITHER of these two logarithms a correction, obtained by multiplying the difference between them by the difference between the given number and the three-figure number which corresponds to the logarithm chosen to be corrected, and rejecting (with due attention to the rule of § 2, e) as many figures at the end of the product as are contained in the latter difference.* The table of **Proportional Parts** may be employed in performing the multiplications. Thus, to find $\log 5668.4$. Using the notation of the formulas of interpolation, and remembering that the place of the decimal-point in the given number may be disregarded in finding the mantissa of the required logarithm, we have

$$\begin{array}{ll} x_1 = 566, & u_1 = \text{mant } \log x_1 = 7528, \\ x_2 = 567, & u_2 = \text{mant } \log x_2 = 7536, \\ \Delta x = 1, & \Delta u = 8; \end{array}$$

so that $\log x$ may be found by either of the following methods: —

$$\lambda = 0.84, \quad \lambda \Delta u = 6.72 = 7 \text{ to units}, \quad u = 7528 + 7 = 7535;$$

or,

$$\lambda' = 1 - \lambda = 0.16, \quad \lambda' \Delta u = 1.28 = 1 \text{ to units}, \quad u = 7536 - 1 = 7535.$$

$$\therefore \log 5668.4 = 3.7535.$$

Let the beginner find the following logarithms by this method: —

$$\begin{array}{ll} \log 59.43 & = 1.7740, & \log 0.0081472 & = 7.9110 - 10, \\ \log 284.8 & = 2.4545, & \log 572820 & = 5.7581, \\ \log 0.073748 & = 8.8678 - 10, & \log 0.50167 & = 9.7004 - 10, \\ \log 3.1607 & = 0.4998, & \log 99968. & = 4.9999. \end{array}$$

The interpolated logarithm should never be carried to more than four decimal-places.

d. The work of interpolation may be shortened by using the column of *proportional parts*, marked P. P., on the right of the table. In using this column, one must work from the three-figure number **NEAREST** to the given

Explanation of the Tables.

number (in the above example, from 567, not from 566). If the given number has only four figures, so that λ or λ' has only one figure, then the correction will be found in the column P. P., under λ or λ' (according as we are working from the number below or the number above the given number), and in the same line with the logarithm to be corrected. If the given number has more than four figures, the correction must be estimated by the observation of the corrections which correspond to the figures below and above the first figure of λ or λ' . E. g.

$$\text{mant log } 2848 = \text{mant log } 285 - \text{cor. for } .2 = 4548 - 3 = 4545;$$

$$\text{mant log } 56684 = \text{mant log } 567 - \text{cor. for } .16 = 7536 - 2 = 7534$$

In the last case the correction is either 1 or 2, and, since .16 is nearer .20 than .10, we choose the correction belonging to .20. Larger tables show that the mantissa of the required logarithm, to five places, is 75346; so that the value found by the column P. P. is here nearly as accurate as that obtained by computation. There is a slightly greater liability to error when we use the column P. P. than when we interpolate by computation; but the disadvantage is generally insignificant. The last figure of an interpolated logarithm obtained from any table may always be one unit in error. E. g.: the true mant log of 57282 to five places is 75802; and this is a case in which the column P. P. gives a better result than computation.

The student is advised now to find all the logarithms in the above list by using the column P. P.

e. If the first figure of the given number is 1, it will be found tabulated to four figures in pp. 4, 5. The correction for a fifth and following figures may be found by the method of interpolation explained in c. As the differences are always small on these pages, and the corrections easily computed, the column P. P. is not here given; but, to facilitate taking the last difference, we have printed at the end of each line, under the heading 10, the first logarithm of the following line. Let the student find the following logarithms:—

$$\begin{array}{ll} \log 11.737 = 1.0696, & \log 0.00100066 = 7.0003 - 10, \\ \log 0.15703 = 9.1960 - 10, & \log 18597. = 4.2694. \end{array}$$

To find the antilogarithm of any logarithm.

f. It is enough to explain the way of finding the series of significant figures which compose the antilogarithm, by means of the mantissa of the given logarithm; the pointing off of the antilogarithm being determined, according to rule, by the given characteristic. If the mantissa of the given logarithm is contained in the table, the required antilogarithm is at once found by inspection. Otherwise, we must resort to the formulas of interpolation, which give the following rule:—Find two successive tabulated logarithms (u_1 and u_2) between which the given logarithm (u) lies; then divide the difference between either of these tabulated logarithms and the given logarithm ($u - u_1$ or $u_2 - u$) by the difference between the tabulated logarithms (Δu), carry out the quotient to the NEAREST tenth (that is, to one figure, which may be 0), and add it to or subtract it from the antilogarithm (x_1 or x_2) of the tabulated logarithm (u_1 or u_2) with which the given logarithm has been compared. The antilogarithm is always a figure annexed to the three or four tabulated figures of x_1 .

Logarithms of Sums and Differences.

The division should not generally be carried beyond one figure. Even the first figure is, in most cases, somewhat uncertain. If the mantissa of the given logarithm is less than 3010, pp. 4, 5 should be used. On pp. 2, 3, the column P. P. may be employed.

Let it be required to find $\log^{-1} 1.5284$. We find

$$u_1 = 5276, \quad x_1 = 337, \quad u - u_1 = 8, \quad u_2 - u = 5,$$

$$u_2 = 5289, \quad x_2 = 338, \quad \mu = \mu \Delta x = \frac{8}{13} = 0.6 \dots,$$

$$\Delta u = 13, \quad \Delta x = 1, \quad \mu' = \mu' \Delta x = \frac{5}{13} = 0.4 \dots,$$

$$x = 337 + 0.6 = 338 - 0.4 = 337.6;$$

$$\therefore \log^{-1} 1.5284 = 33.76.$$

More briefly, looking along the line of $u_2 = 5289$ for $5 = u_2 - u$ in column P. P., we find that 5 corresponds to the correction 4, which gives at once the required number. In like manner, the student may find

$$\log^{-1} 1.9155 = 82.32, \quad \log^{-1} (5.8760 - 10) = 0.00007517,$$

$$\log^{-1} 3.8291 = 6747, \quad \log^{-1} (9.5727 - 10) = 0.3738,$$

$$\log^{-1} 0.1548 = 1.4283, \quad \log^{-1} (8.2731 - 10) = 0.018755.$$

g. The convenient usage of making negative characteristics positive, by the addition of 10, is followed, throughout the present collection of tables, whenever logarithms are printed with their characteristics. This must be always understood, though no explicit reference be made to it in the explanation of the table.

§ 5. LOGARITHMS OF SUMS AND DIFFERENCES.

a. This is one form of a table devised by GAUSS to facilitate finding the logarithm of the *sum* or *difference* of two numbers which are themselves given only by their logarithms. The *argument* of the table is any logarithm, and may be called $\log x$; the *function* tabulated is then $\log (x + 1)$. It follows that, if the function is denoted by $\log x$, the argument is $\log (x - 1)$. The function may be called the **Gaussian** of the argument, and the argument the **anti-Gaussian** of the function; and the symbols \mathcal{G} and \mathcal{G}^{-1} may be used to denote these relations. Thus we have

$$\log (x + 1) = \mathcal{G} \log x, \quad \log (x - 1) = \mathcal{G}^{-1} \log x.$$

b. To find the **Gaussian** of a given logarithm. Seek the characteristic of the given logarithm (increased by 10 if negative) at the top of the table, and the first two figures of the mantissa in the left-hand column. If the third and fourth figures of the mantissa are zero, the Gaussian will be found in the column and line thus determined; otherwise, it can be obtained by the method of interpolation which has been fully explained in § 2. In three columns of the table, the *rate of difference* of the Gaussian is printed in small type after the value of the function, and may be used instead of the tabular difference of the Gaussian through half the tabular interval before and after the value to which it is attached, as explained in

Explanation of the Tables.

§ 2, c, and completely illustrated below, in the explanation of the table of *Logarithms of Circular Functions*. The table of Proportional Parts may be employed in computing the corrections. Examples:—

$$\begin{array}{ll} \textcircled{G} 1.0960 = 1.1295, & \textcircled{G} (7.5265 - 10) = 0.0015, \\ \textcircled{G} 3.8129 = 3.8130, & \textcircled{G} (9.6431 - 10) = 0.1582. \end{array}$$

$$\begin{array}{ll} \text{If } \log x < 6.0000 - 10, & \log (x + 1) = 0.0000 \text{ to four places;} \\ \text{if } \log x > 4.0000, & \log (x + 1) = \log x \text{ to four places.} \end{array}$$

c. **To find the anti-Gaussian of a given logarithm.** Seek, in the body of the table, two successive logarithms between which the given logarithm lies, and then find the corresponding value of the argument by interpolation. Examples:—

$$\begin{array}{ll} \textcircled{G}^{-1} 1.0960 = 1.0597, & \textcircled{G}^{-1} 0.1051 = 9.4373 - 10, \\ \textcircled{G}^{-1} 3.8129 = 3.8128, & \textcircled{G}^{-1} 1.0216 = 0.9782. \end{array}$$

d. **To find the logarithm of the sum or difference of the anti-logarithms of two given logarithms.** If m and n are two numbers,

$$m + n = n \left(\frac{m}{n} + 1 \right), \quad m - n = n \left(\frac{m}{n} - 1 \right),$$

$$\log (m + n) = \log n + \log \left(\frac{m}{n} + 1 \right) = \log n + \textcircled{G} \log \frac{m}{n},$$

$$\log (m - n) = \log n + \log \left(\frac{m}{n} - 1 \right) = \log n + \textcircled{G}^{-1} \log \frac{m}{n}.$$

Example:—

Given $a = 4.142$, $b = 2.399$; to find $\sqrt{a^2 + b^2}$ and $\sqrt{a^2 - b^2}$.

$$\begin{array}{llll} \log a & = 0.6172, & \log b & = 0.3800, \\ \log a^2 & = 1.2344, & \log b^2 & = 0.7600, \\ \log \frac{a^2}{b^2} & = 0.4744; & & \\ \textcircled{G} \log \frac{a^2}{b^2} & = 0.6000, & \textcircled{G}^{-1} \log \frac{a^2}{b^2} & = 0.2970, \\ \log b^2 & = 0.7600, & \log b^2 & = 0.7600, \\ \log (a^2 + b^2) & = 1.3800, & \log (a^2 - b^2) & = 1.0570, \\ \log \sqrt{a^2 + b^2} & = 0.6800, & \log \sqrt{a^2 - b^2} & = 0.5285, \\ \sqrt{a^2 + b^2} & = 4.787; & \sqrt{a^2 - b^2} & = 3.377. \end{array}$$

§ 6. CIRCULAR, OR TRIGONOMETRIC, FUNCTIONS: NATURAL VALUES.

a. Three tables of the natural values of the trigonometric functions are given on pp. 22-27. Each table is broken up into six divisions, and occupies two pages. The argument is the angle, which is tabulated at intervals of 10' from 0° to 90°. Angles in the *first half of the quadrant* will be found in the *left-hand column* of the several divisions of the table, and for those angles the names of the functions are to be taken from the top of the page; angles in the *second half of the quadrant* are to be found in the right-hand

Logarithms of Circular Functions.

column of the table, and for those angles the names of the functions are to be taken from the *bottom* of the page. The angles standing at the right and left in the same line are *complements* of each other; and the names of the functions at the top and bottom of the same column are *complementary*. The value of any of the functions for a non-tabulated angle, or the value of the angle for a non-tabulated value of one of the functions, can be found by the method of interpolation explained in § 2. The precepts of § 2, *d*, *e*, should be observed in computing the corrections. The tabulated values of the functions are generally given to four significant figures; but, in the tables of tangents and secants, they are sometimes given to a less number of figures (to avoid errors in interpolation), and are sometimes omitted altogether. In these cases the functions can be best found by finding their logarithms by the table of *Logarithms of Circular Functions* (see § 7), and then the numbers corresponding by the table of *Logarithms*.

b. To find any function of an angle greater than 90° , we must subtract from the given angle the greatest multiple of 90° which it contains; if an *even* multiple has been subtracted, we look out the required function of the remainder; if an *odd* multiple, the complementary function; and we then fix the *sign* of the function by considering the *quadrant* in which the given angle lies. For a *negative* angle, we find the required function of the corresponding positive angle, and then fix its sign by considering the quadrant of the angle.

c. Examples of the use of these tables:—

$$\begin{array}{lll} \sin 77^\circ 37' = 0.9767, & \tan 53^\circ 04' = 1.330, & \sec 68^\circ 45' = 2.759, \\ \cos 16^\circ 19' = 0.9597, & \cot 3^\circ 18' = 17.4, & \csc 55^\circ 13' = 1.217; \end{array}$$

$$\begin{array}{lll} \sin 257^\circ 37' = -0.9767, & \tan 93^\circ 18' = -17.4, & \sec 325^\circ 13' = 1.217, \\ \cos 163^\circ 41' = -0.9597, & \cot 323^\circ 04' = -1.330, & \csc 168^\circ 45' = 2.759; \end{array}$$

$$\begin{array}{lll} \sin (-257^\circ 37') = 0.9767, & \tan (-93^\circ 18') = 17.4, & \sec (-325^\circ 13') = 1.217, \\ \cos (-163^\circ 41') = -0.9597, & \cot (-323^\circ 04') = 1.330, & \csc (-168^\circ 45') = -2.759; \end{array}$$

$$\begin{array}{lll} \sin^{-1} 0.2000 & = 11^\circ 32' \text{ or } = 168^\circ 28' \text{ or } = 371^\circ 32', \text{ etc.,} \\ \cos^{-1} (-0.3542) & = 110^\circ 45' \text{ or } = 249^\circ 15' \text{ or } = 830^\circ 45', \text{ etc.,} \\ \tan^{-1} (-4.570) & = 102^\circ 21' \text{ or } = 282^\circ 21' \text{ or } = -77^\circ 39', \text{ etc.,} \\ \cot^{-1} 0.3163 & = 72^\circ 27' \text{ or } = 252^\circ 27' \text{ or } = -107^\circ 33', \text{ etc.,} \\ \sec^{-1} 5.000 & = 78^\circ 28' \text{ or } = -78^\circ 28' \text{ or } = \pm 281^\circ 32', \text{ etc.,} \\ \csc^{-1} (-3.529) & = -16^\circ 28' \text{ or } = 196^\circ 28' \text{ or } = -163^\circ 32', \text{ etc.} \end{array}$$

§ 7. LOGARITHMS OF CIRCULAR FUNCTIONS.

To find the logarithm of any circular function of a given angle.

a. If the angle is less than 6° , the part of the table which occupies the upper half of p. 10 may be used. (See also *g*.) The *left-hand division* of this part of the table gives the values of a logarithm *S* (the characteristic and the first two figures of the mantissa being printed at the head of the column), with the angular limits between which each value may be used. Thus, for all positive angles less than $1^\circ 51'.479$, $S = 6.4637$; for all angles between $1^\circ 51'.479$ and $2^\circ 49'.567$, $S = 6.4636$; etc. The next following

Explanation of the Tables.

division gives, in like manner, the values of a logarithm T. We must find the logarithm of the angle, reduced to minutes and decimals of a minute, and must then apply the formulas:—

$$\log \sin \phi = \log (\phi \text{ in minutes}) + S - 10,$$

$$\log \tan \phi = \log (\phi \text{ in minutes}) + T - 10.$$

The two right-hand divisions of this part of the table give the values of the log sec, with the angular limits for each value. The logarithms of the co-sine, cotangent, and cosecant are the *arithmetical complements* (— 10) of the logarithms of the secant, tangent, and sine, respectively. Example:—

$$\begin{aligned} 1 \sin 3^\circ 15' .23 &= 8.7541, & 1 \tan 3^\circ 15' .23 &= 8.7548, & 1 \sec 3^\circ 15' .23 &= 0.0007, \\ 1 \csc 3^\circ 15' .23 &= 1.2459, & 1 \cot 3^\circ 15' .23 &= 1.2452, & 1 \cos 3^\circ 15' .23 &= 9.9993; \end{aligned}$$

the negative characteristics being here, as in the following examples, made positive by the addition of 10.

b. If the angle is *acute and greater than 84°*, we must take its complement, and then seek the *function complementary to that required*, for the angle thus obtained, by the method just expounded. Example:—

$$\begin{aligned} 1 \sin 86^\circ 44' .77 &= 9.9993, & 1 \tan 86^\circ 44' .77 &= 1.2452, & 1 \sec 86^\circ 44' .77 &= 1.2459, \\ 1 \csc 86^\circ 44' .77 &= 0.0007, & 1 \cot 86^\circ 44' .77 &= 8.7548, & 1 \cos 86^\circ 44' .77 &= 8.7541. \end{aligned}$$

c. If the angle is contained *between 6° and 84°*, we use the main part of the table, occupying the lower half of p. 10 and pp. 11–15. The angle is tabulated at intervals of 10', from 6° to 45° in the left-hand column of the table, and from 45° to 84° in the right-hand column. The names of the functions are to be taken from the *tops* of the columns, when the angle is on the *left*; and from the *bottoms* of the columns, when the angle is on the *right*. The angles on the right and left of any line and the names at the top and bottom of any column have the same relation to each other as in the tables of *Natural Values* (§ 6). The true characteristic in the first, third, and sixth columns is —1, but is printed 9. The six columns are arranged in pairs. The two functions in each pair of columns are *reciprocal* to each other; and the logarithms are therefore *complementary*, and their differences are equal in value, with opposite signs. Down the middle of each double column are printed, in small type, the *rates of difference* of the logarithms in that double column. Each value of this rate may be used in interpolation, instead of Δu , through half the interval before and after the line on which it stands, as stated in § 2, c. Thus, in finding the logarithms of the circular functions of any angle between 25° 25' and 25° 35' we work from the values corresponding to 25° 30', the nearest tabulated angle; and compute the corrections by taking proportional parts of 26, 33, and 6, for the three pairs of functions. In applying the corrections, we must carefully observe, for each function, whether the function is increasing or decreasing.

For example, let the logarithms of the circular functions of 25° 27' .4 = 25° 30' — 02' .6 be required. We find

$$\begin{array}{lll} 1 \sin 25^\circ 30' &= 9.6340 & 1 \tan 25^\circ 30' &= 9.6785 & 1 \sec 25^\circ 30' &= 0.0445 \\ 0.26 \times 26 &= 7 & 0.26 \times 33 &= 9 & 0.26 \times 6 &= 2 \\ 1 \sin 25^\circ 27' .4 &= 9.6333, & 1 \tan 25^\circ 27' .4 &= 9.6776, & 1 \sec 25^\circ 27' .4 &= 0.0443. \end{array}$$

Logarithms of Circular Functions.

$1 \csc 25^\circ 30' = 0.3660$	$1 \csc 25^\circ 30' = 0.3660$	$1 \csc 25^\circ 30' = 0.3660$
7	9	2
$1 \csc 25^\circ 27'.4 = 0.3667,$	$1 \csc 25^\circ 27'.4 = 0.3667,$	$1 \csc 25^\circ 27'.4 = 0.3667.$

In like manner, we have

$1 \sin 74^\circ 46' = 9.9845,$	$1 \tan 74^\circ 46' = 0.5649,$	$1 \sec 74^\circ 46' = 0.5804,$
$1 \csc 74^\circ 46' = 0.0155,$	$1 \csc 74^\circ 46' = 0.0155,$	$1 \csc 74^\circ 46' = 0.0155.$

d. If the angle is greater than 90° , or negative, we must use the method explained in § 6, b, for the tables of *Natural Values* of the circular functions. When the natural value of a circular function is negative, this should be indicated by writing the letter *n* after its logarithm. Examples:

$1 \sin 105^\circ 14' = 9.9845,$	$1 \tan 105^\circ 14' = 0.5649 n,$
$1 \sec 105^\circ 14' = 0.5804 n,$	
$1 \csc 164^\circ 46' = 0.5804,$	$1 \csc 164^\circ 46' = 0.5649 n,$
$1 \cos 164^\circ 46' = 9.9845 n,$	
$1 \sin (-74^\circ 46') = 9.9845 n,$	$1 \tan (-105^\circ 14') = 0.5649,$
$1 \cos (-394^\circ 46') = 9.4196.$	

Given the logarithm of any circular function, to find the value of the corresponding angle.

e. If the given logarithm lies *without the limits of the main part of the table*, the upper part of p. 10 may be used. If the given logarithm is a *log sin less than 9.0192*, or a *log tan less than 9.0216*, subtract from it the proper value of S or T (or add the arithmetical complement), and the remainder is the log of the required angle in minutes. The limiting values of the log sin and log tan for each value of S and T are given in the table. If the given log is a *log csc greater than 0.9808*, or a *log ctn greater than 0.9784*, its arithmetical complement will be a log sin less than 9.0192, or a log tan less than 9.0216. If the given log is a *log sec less than 0.0024*, the limits between which the required angle lies are given by the table; the angle may have any value between these limits, and is not therefore very closely determined. If the given log is a *log cos greater than 9.9976*, its arithmetical complement is a log sec less than 0.0024.

If the given log is a *log sin, log tan, or log sec greater than 9.9976, 0.9784, or 0.9808* (respectively), or a *log csc, log ctn, or log cos less than 0.0024, 9.0216, or 9.0192* (respectively), we must change the name of the function to the *complementary name* (sin to cos, etc.), find the corresponding angle as above, and take the complement of the angle thus found. Examples:—

$(\log \sin)^{-1} 8.9542 = 5^\circ 09'.8,$	$(\log \csc)^{-1} 2.0531 = 0^\circ 30'.42,$
$(\log \tan)^{-1} 9.0024 = 84^\circ 15'.5,$	$(\log \sec)^{-1} 9.9983 = 84^\circ 56' \pm 4'.$

f. If the given logarithm is contained *within the limits of the main part of the table*, the required angle is found by ordinary interpolation; and we may use the printed rate of difference as the value of Δu , working in each case from the nearest tabulated value. The angle should be found to the nearest minute, or, when the difference exceeds 100, to the nearest tenth of a

Explanation of the Tables.

minute. But in the right-hand pair of columns, the last figure of the angle thus found will generally be uncertain. Examples:—

Let it be required to find $(\log \sec)^{-1} 0.0343$; i. e. the angle of which the log sec is 0.0643. The nearest tabulated log sec is 0.0647. We have, then,

$$(\log \sec)^{-1} 0.0647 = 30^\circ 30', u_2 - u = 4, \Delta u = 7, \frac{4}{7} = 0.6,$$

$$\therefore (\log \sec)^{-1} 0.0643 = 30^\circ 30' - 08' = 30^\circ 24'.$$

In like manner, let the student find

$$(\log \sin)^{-1} 9.5663 = 21^\circ 37',$$

$$(\log \cot)^{-1} 0.0496 = 41^\circ 44',$$

$$(\log \cos)^{-1} 9.9188 = 33^\circ 58' \text{ or } 57',$$

$$(\log \sec)^{-1} 0.2272 = 53^\circ 39',$$

$$(\log \tan)^{-1} 0.7507 = 79^\circ 56',$$

$$(\log \csc)^{-1} 0.1433 = 45^\circ 58'.$$

The angle may also be found by the next following table.

g. Pp. 8 and 9 may also be used for angles less than 6° or greater than 84° .

E.g. $1 \sin 4^\circ 03'.4 = 8.8497$, $1 \tan 4^\circ 03'.4 = 8.8508$, $1 \sec 4^\circ 03'.4 = 0.0011$,

$1 \csc 4^\circ 03'.4 = 1.1503$, $1 \cot 4^\circ 03'.4 = 1.1492$, $1 \cos 4^\circ 03'.4 = 9.9989$.

§ 8. INVERSE CIRCULAR FUNCTIONS.

a. The table having this heading (pp. 16–18) is a table for finding the angle which corresponds to the given logarithm of a circular function. The logarithm (increased by 10) is the *argument* of the table, and is to be regarded as given to *four places* of decimals. It is tabulated at intervals of 0.0100 from 9.0000 to 0.0000 through the first page of the table, then at intervals of 0.0010, and in the last two divisions at intervals of 0.0001. The characteristic of the argument is printed at the head of the column. The figures supposed to follow the printed figures in the values of the argument are zeros. Thus, the first value is 9.0000, the next 9.0100, etc. The angle is given, for convenience of interpolation, in degrees and decimals of a degree. When found, it is easily reduced to degrees and minutes, if that is necessary, and should, in general, be taken *only to the nearest minute*. The angle under the heading $\sin^{-1} u$ is that angle of which the corresponding value of the argument, $\log u$, is the $\log \sin$; etc. In interpolating in this table, we may use the *printed rate of difference* instead of Δu , working from the *nearest* tabulated value of the argument, and carefully observing whether the tabulated angle ought to be *increased or diminished*. When the *printed rate of difference is omitted*, this is because the interval is too great to admit of accurate interpolation. In this case, we must resort to those later divisions of the table in which the argument is tabulated at smaller intervals. When the last figure of the tabulated angle is printed in small type, this shows that that figure is uncertain, if the logarithm is given to only four places; that is, that there is a possible variation, on each side of the tabulated angle, as great as *half a unit* in the place of the figure so printed. For example, if $\log u = 9.9000$, we find the last figures of $\sin^{-1} u$ and $\cos^{-1} u$ to be printed in small type. Now, seven-place tables show that $(\log \sin)^{-1} 9.8999500 = 52^\circ.581$, while $(\log \sin)^{-1} 9.9000500 = 52^\circ.600$. But 9.9000 may represent any logarithm between these; and hence the corresponding angle, in this case, admits a like variation, while $\cos^{-1} u$ may have any value between $37^\circ.419$ and $37^\circ.400$.

Hyperbolic Functions.

Neither of these difficulties presents itself in finding an angle from its $\log \tan$ or $\log \csc$. If $\log u = 9.9000$, $\tan^{-1} u$ can only vary from $38^{\circ}.458$ to $38^{\circ}.464$.

The angle found by interpolation should be carried out *only to the nearest hundredth of a degree, in any case*. The last column of the table shows that the angle is not always determined even to the nearest *tenth*.

b. If the characteristic of the given logarithm is 0, we must take its arithmetical complement, which will be the logarithm of the reciprocal function of the same angle. The angle can then be found by the table.

c. If the given logarithm is less than 9.0000, or greater than 0.0000, the tables in the upper part of p. 10 may be used, as explained in § 7, e; or pp. 8, 9.

d. Let us find by this table the angles sought above, in § 7, f. We have, in the case of the first example,

$$(\log \sec)^{-1} 0.0643 = (\log \cos)^{-1} 9.9357.$$

Then the table gives

$$(\log \cos)^{-1} 9.9360 = 30^{\circ}.35, \quad \Delta u = 23,$$

$$0.3 \times 0.23 = .07$$

$$\therefore (\log \cos)^{-1} 9.9357 = 30^{\circ}.42 = 30^{\circ} 25'.$$

In fact, the limits of the angle are $30^{\circ} 24'.2$ and $30^{\circ} 25'.6$, the mean value being $30^{\circ} 24'.9$. In this case, the present table gives a better value than the other; but both values are admissible.

In like manner, we have

$$(\log \sin)^{-1} 9.5663 = 21^{\circ}.81 - 20 = 21^{\circ}.61 = 21^{\circ} 37',$$

$$(\log \cos)^{-1} 9.9188 = 33^{\circ}.92 + .04 = 33^{\circ}.96 = 33^{\circ} 58',$$

$$(\log \tan)^{-1} 0.7507 = 79^{\circ}.92 + .02 = 79^{\circ}.94 = 79^{\circ} 56',$$

$$(\log \csc)^{-1} 0.0496 = 41^{\circ}.71 + .03 = 41^{\circ}.74 = 41^{\circ} 44',$$

$$(\log \sec)^{-1} 0.2272 = 53^{\circ}.64 + .02 = 53^{\circ}.66 = 53^{\circ} 40',$$

$$(\log \csc)^{-1} 0.1433 = 46^{\circ}.01 - .04 = 45^{\circ}.97 = 45^{\circ} 58'.$$

§ 9. HYPERBOLIC FUNCTIONS.

a. The **hyperbolic functions** are certain functions which bear relations to the equilateral hyperbola similar to those borne by the circular functions to the circle; and they may often be usefully employed both in computation and in analysis. They are named the hyperbolic sine, cosine, tangent, cotangent, secant, and cosecant; and are variously denoted by different writers. They are here represented by the symbols: Sh, Ch, Th, Cth, Sch, Csch. They may be defined by the following formulas, in which

G = the exponential base

$$= 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \dots$$

$$= 2.7182818285 \dots :-$$

$$\text{Sh } x = \frac{1}{2} (G^x - G^{-x}), \quad \text{Ch } x = \frac{1}{2} (G^x + G^{-x}), \quad \text{Th } x = \frac{\text{Sh } x}{\text{Ch } x},$$

$$\text{Cth } x = \frac{1}{\text{Th } x}, \quad \text{Sch } x = \frac{1}{\text{Ch } x}, \quad \text{Csch } x = \frac{1}{\text{Sh } x}.$$

Explanation of the Tables.

They bear to the circular functions the relations expressed by the following formulas, in which $i = \sqrt{-1}$:—

$$\begin{array}{ll}
 \text{Sh } x = \frac{\sin xi}{i}, & \sin x = \frac{\text{Sh } xi}{i}, \\
 \text{Ch } x = \cos xi, & \cos x = \text{Ch } xi, \\
 \text{Th } x = \frac{\tan xi}{i}, & \tan x = \frac{\text{Th } xi}{i}, \\
 \text{Cth } x = i \text{ctn } xi, & \text{ctn } x = i \text{Cth } xi, \\
 \text{Sch } x = \sec xi, & \sec x = \text{Sch } xi, \\
 \text{Csch } x = i \csc xi, & \csc x = i \text{Csch } xi.
 \end{array}$$

Again, if ϕ is so taken that

$$x = \text{nat log tan } (45^\circ + \frac{1}{2} \phi),$$

$$\begin{array}{lll}
 \text{Sh } x = \tan \phi, & \text{Ch } x = \sec \phi, & \text{Csch } x = \text{ctn } \phi, \\
 \text{Th } x = \sin \phi, & \text{Sch } x = \cos \phi, & \text{Cth } x = \csc \phi.
 \end{array}$$

The value of ϕ determined by this formula has been called by some writers the **Gudermannian** of x , and denoted by the symbol: $\text{gd } x$.

b. From $x = 0.00$ to $x = 1.00$, the function tabulated is $\text{gd } x$ in degrees, at intervals of 0.01 in the value of x . The hyperbolic functions of x are then readily found, by the aid of the formulas last given, from the tables of circular functions. Beginning with $x = 1.00$, $\log \text{Sh } x$, $\log \text{Ch } x$, and $\log \text{Th } x$ are tabulated, at intervals of 0.01 in the value of x , up to $x = 3.00$, the characteristic of each logarithm being placed at the head of its column; then at intervals of 0.1 up to $x = 6.0$; and lastly at intervals of 1 up to $x = 10.0$. The printed differences are to be used, as in other tables, each through *half the interval* before and after the line on which it stands.

c. If $x > 10$, $\log \text{Th } x = 0.0000$, while $\log \text{Sh } x$ and $\log \text{Ch } x$ may be found by the formula and table given at the lower right-hand corner of p. 21. The quantity μ is the *modulus* of the denary system of logarithms; that is, it is the denary logarithm of the exponential base. The values of $n\mu$ being given for all integral values of n from 1 to 10, any product $x\mu$ is readily found, by adding together the products of μ by the successive figures of x . Only four decimal-figures should be retained in the result.

d. The functions $\log \text{Cth } x$, $\log \text{Sch } x$, and $\log \text{Csch } x$ are the *arithmetical complements* of $\log \text{Th } x$, $\log \text{Ch } x$, and $\log \text{Sh } x$, respectively.

e. The table may be used both directly and inversely. Examples:—

$$\begin{array}{ll}
 \log \text{Sh } 0.5378 & = \log \tan 29^\circ.427 = 9.7513, \\
 \log \text{Ch } 1.5280 & = 0.3825, \\
 \log \text{Cth } 1.8240 & = 0.0226, \\
 \log \text{Sh } 12.5913 & = \log \text{Ch } 12.5913 = 5.1673, \\
 (\log \text{Ch})^{-1} 1.6000 & = 4.377, \\
 (\log \text{Sh})^{-1} 5.0000 & = 12.206, \\
 (\log \text{Th})^{-1} 9.9012 & = 1.089, \\
 (\log \text{Sch})^{-1} 9.5873 & = 1.604, \\
 (\log \text{Csch})^{-1} 0.3924 & = \text{gd}^{-1} (\log \tan)^{-1} 9.6076 \\
 & = \text{gd}^{-1} 22^\circ.06 = 0.396.
 \end{array}$$

Natural Logarithms

§ 10. NATURAL LOGARITHMS.

a. The **natural system** of logarithms is that which is founded on the **exponential base** (see § 9). This number is defined as *the limiting value to which the expression*

$$(1 + \epsilon)^{\frac{1}{\epsilon}} = \sqrt[\epsilon]{1 + \epsilon}$$

approaches, as ϵ approaches 0. It is most frequently denoted by the letter e ; but, as being one of the few peculiar constants of analysis, it is here represented by the symbol \mathcal{G} , which may be read "**base**."

The following formulas are proved in treatises on the Differential Calculus:—

$$\mathcal{G} = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \dots,$$

$$\mathcal{G}^x = 1 + \frac{x}{1} + \frac{x^2}{1.2} + \frac{x^3}{1.2.3} + \frac{x^4}{1.2.3.4} + \dots,$$

$$\text{nat log } (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots;$$

the second formula being applicable to *all values* of x , but the last only when x is *numerically less than 1*. If x is *very small*, then *approximately*

$$\mathcal{G}^x = 1 + x, \quad \text{nat log } (1 + x) = x, \quad \text{nat log } (1 - x) = -x.$$

We also have, in the natural system,

$$\log (a + h) = \log a + \log \left(1 + \frac{h}{a} \right) = \log a + \frac{h}{a} - \frac{h^2}{2a^2} + \frac{h^3}{3a^3} - \dots,$$

provided h is numerically less than a .

The *rate of difference* of \mathcal{G}^x , for $\Delta x = 1$, is always \mathcal{G}^x , and that of $\text{nat log } x$ is $\frac{1}{x}$.

b. The **numerical value** of \mathcal{G} or of any power of \mathcal{G} can be computed, to *any assigned number of decimal-places*, by using a sufficient number of terms of the first two series given above. Thus, to find \mathcal{G} to four decimal-places, we proceed as follows, observing that, if any term be divided by its number in the series, the next following term is obtained:—

$$\begin{array}{r} 1) 1.00000 \\ 2) 1.00000 \\ 3) 0.50000 \\ 4) 0.16667 \\ 5) 0.04167 \\ 6) 0.00833 \\ 7) 0.00139 \\ 8) 0.00020 \\ \quad 0.00002 \\ \hline \mathcal{G} = 2.7183 \dots \end{array}$$

Explanation of the Tables.

They bear to the circular functions the relations expressed by the following formulas, in which $i = \sqrt{-1}$:—

$$\begin{array}{ll} \text{Sh } x = \frac{\sin xi}{i}, & \sin x = \frac{\text{Sh } x}{i}, \\ \text{Ch } x = \cos xi, & \cos x = \frac{\text{Ch } x}{i}, \\ \text{Th } x = \frac{\tan xi}{i}, & \tan x = \frac{\text{Th } x}{i}, \\ \text{Cth } x = i \operatorname{ctn} xi, & \operatorname{ctn} x = \frac{\text{Cth } x}{i}, \\ \text{Sch } x = \sec xi, & \sec x = \frac{\text{Sch } x}{i}, \\ \text{Csch } x = i \operatorname{csc} xi, & \operatorname{csc} x = \frac{\text{Csch } x}{i}. \end{array}$$

Again, if ϕ is so taken that

$$x = \text{nat log tan } (45^\circ + \tfrac{1}{2} \phi),$$

$$\begin{array}{lll} \text{Sh } x = \tan \phi, & \text{Ch } x = \sec \phi, & \text{Cth } x = \operatorname{ctn} \phi, \\ \text{Th } x = \sin \phi, & \text{Sch } x = \cos \phi, & \text{Csch } x = \operatorname{csc} \phi. \end{array}$$

The value of ϕ determined by this formula has been called the **Gudermannian** of x , and denoted by the symbol ϕ .

b. From $x = 0.00$ to $x = 1.00$, the function tabulated at intervals of 0.01 in the value of x . The hyperbolic functions are then readily found, by the aid of the formulas last given, from the values of circular functions. Beginning with $x = 1.00$, $\log \text{Th } x$ and $\log \text{Sch } x$ are tabulated, at intervals of 0.01 in the value of x , the characteristic of each logarithm being placed in the third column; then at intervals of 0.1 up to $x = 6.0$; and lastly up to $x = 10.0$. The printed differences are to be used each through *half the interval* before and after the line of the table.

c. If $x > 10$, $\log \text{Th } x = 0.0000$, while $\log \text{Sch } x$ are found by the formula and table given at the lower right of p. 21. The quantity μ is the *modulus* of the denary system; that is, it is the denary logarithm of the exponential base e , $n\mu$ being given for all integral values of n from 1 to 10. The values of $\log \text{Th } x$ and $\log \text{Sch } x$ are readily found, by adding together the products of μ and the corresponding figures of x . Only four decimal-figures should be retained.

d. The functions $\log \text{Cth } x$, $\log \text{Sch } x$, and $\log \text{Csch } x$ are the *complements* of $\log \text{Th } x$, $\log \text{Ch } x$, and $\log \text{Sh } x$, respectively.

e. The table may be used both directly and inversely.

$$\begin{array}{ll} \log \text{Sh } 0.5378 & = \log \tan 29^\circ.427 \\ \log \text{Ch } 1.5280 & = 0.3825, \\ \log \text{Cth } 1.8240 & = 0.0226, \\ \log \text{Sh } 12.5913 & = \log \text{Ch } 12.5913 \\ (\log \text{Ch})^{-1} 1.6000 & = 4.377, \\ (\log \text{Sh})^{-1} 5.0000 & = 12.206, \\ (\log \text{Th})^{-1} 9.9012 & = 1.089, \\ (\log \text{Sch})^{-1} 9.5873 & = 1.604, \\ (\log \text{Csch})^{-1} 0.3924 & = \operatorname{gd}^{-1} (\log \tan)^{-1} 22^\circ.06 \\ & = \operatorname{gd}^{-1} 22^\circ.06 = 0.395. \end{array}$$

15					20				
1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	1	
2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	2	
3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0	3	
5.2	5.6	6.0	6.4	6.8	7.2	7.6	8.0	4	
6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	5	
7.8	8.4	9.0	9.6	10.2	10.8	11.4	12.0	6	
9.1	9.8	10.5	11.2	11.9	12.6	13.3	14.0	7	
10.4	11.2	12.0	12.8	13.6	14.4	15.2	16.0	8	
11.7	12.6	13.5	14.4	15.3	16.2	17.1	18.0	9	

35					40				
3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	1	
6.6	6.8	7.0	7.2	7.4	7.6	7.8	8.0	2	
9.9	10.2	10.5	10.8	11.1	11.4	11.7	12.0	3	
13.2	13.6	14.0	14.4	14.8	15.2	15.6	16.0	4	
16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0	5	
19.8	20.4	21.0	21.6	22.2	22.8	23.4	24.0	6	
23.1	23.8	24.5	25.2	25.9	26.6	27.3	28.0	7	
26.4	27.2	28.0	28.8	29.6	30.4	31.2	32.0	8	
29.7	30.6	31.5	32.4	33.3	34.2	35.1	36.0	9	

55					60				
5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	1	
10.6	10.8	11.0	11.2	11.4	11.6	11.8	12.0	2	
15.9	16.2	16.5	16.8	17.1	17.4	17.7	18.0	3	
21.2	21.6	22.0	22.4	22.8	23.2	23.6	24.0	4	
26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	5	
31.8	32.4	33.0	33.6	34.2	34.8	35.4	36.0	6	
37.1	37.8	38.5	39.2	39.9	40.6	41.3	42.0	7	
42.4	43.2	44.0	44.8	45.6	46.4	47.2	48.0	8	
47.7	48.6	49.5	50.4	51.3	52.2	53.1	54.0	9	

75					80				
7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0	1	
14.6	14.8	15.0	15.2	15.4	15.6	15.8	16.0	2	
21.9	22.2	22.5	22.8	23.1	23.4	23.7	24.0	3	
29.2	29.6	30.0	30.4	30.8	31.2	31.6	32.0	4	
36.5	37.0	37.5	38.0	38.5	39.0	39.5	40.0	5	
43.8	44.4	45.0	45.6	46.2	46.8	47.4	48.0	6	
51.1	51.8	52.5	53.2	53.9	54.6	55.3	56.0	7	
58.4	59.2	60.0	60.8	61.6	62.4	63.2	64.0	8	
65.7	66.6	67.5	68.4	69.3	70.2	71.1	72.0	9	

95					100				
9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0	1	
18.6	18.8	19.0	19.2	19.4	19.6	19.8	20.0	2	
27.9	28.2	28.5	28.8	29.1	29.4	29.7	30.0	3	
37.2	37.6	38.0	38.4	38.8	39.2	39.6	40.0	4	
46.5	47.0	47.5	48.0	48.5	49.0	49.5	50.0	5	
55.8	56.4	57.0	57.6	58.2	58.8	59.4	60.0	6	
65.1	65.8	66.5	67.2	67.9	68.6	69.3	70.0	7	
74.4	75.2	76.0	76.8	77.6	78.4	79.2	80.0	8	
83.7	84.6	85.5	86.4	87.3	88.2	89.1	90.0	9	

Explanation of the Tables.

§ 2, c, and completely illustrated below, in the explanation of the table of *Logarithms of Circular Functions*. The table of Proportional Parts may be employed in computing the corrections. Examples:—

$$\mathfrak{G} 1.0960 = 1.1295,$$

$$\mathfrak{G} 3.8129 = 3.8130,$$

$$\mathfrak{G} (7.5265 - 10) = 0.0015,$$

$$\mathfrak{G} (9.6431 - 10) = 0.1582.$$

$$\text{If } \log x < 6.0000 - 10,$$

$$\text{if } \log x > 4.0000,$$

$$\log (x + 1) = 0.0000 \text{ to four places ;}$$

$$\log (x + 1) = \log x \text{ to four places.}$$

c. **To find the anti-Gaussian of a given logarithm.** Seek, in the body of the table, two successive logarithms between which the given logarithm lies, and then find the corresponding value of the argument by interpolation. Examples:—

$$\mathfrak{G}^{-1} 1.0960 = 1.0597,$$

$$\mathfrak{G}^{-1} 3.8129 = 3.8128,$$

$$\mathfrak{G}^{-1} 0.1051 = 9.4373 - 10,$$

$$\mathfrak{G}^{-1} 1.0216 = 0.9782.$$

d. **To find the logarithm of the sum or difference of the anti-logarithms of two given logarithms.** If m and n are two numbers,

$$m + n = n \left(\frac{m}{n} + 1 \right), \quad m - n = n \left(\frac{m}{n} - 1 \right),$$

$$\log (m + n) = \log n + \log \left(\frac{m}{n} + 1 \right) = \log n + \mathfrak{G} \log \frac{m}{n},$$

$$\log (m - n) = \log n + \log \left(\frac{m}{n} - 1 \right) = \log n + \mathfrak{G}^{-1} \log \frac{m}{n}.$$

Example:—

Given $a = 4.142$, $b = 2.399$; to find $\sqrt{a^2 + b^2}$ and $\sqrt{a^2 - b^2}$.

$$\log a = 0.6172,$$

$$\log a^2 = 1.2344,$$

$$\log \frac{a^2}{b^2} = 0.4744;$$

$$\log b = 0.3800,$$

$$\log b^2 = 0.7600,$$

$$\mathfrak{G} \log \frac{a^2}{b^2} = 0.6000,$$

$$\log b^2 = 0.7600,$$

$$\log (a^2 + b^2) = 1.3600,$$

$$\log \sqrt{a^2 + b^2} = 0.6800,$$

$$\sqrt{a^2 + b^2} = 4.787;$$

$$\mathfrak{G}^{-1} \log \frac{a^2}{b^2} = 0.2970,$$

$$\log b^2 = 0.7600,$$

$$\log (a^2 - b^2) = 1.0570,$$

$$\log \sqrt{a^2 - b^2} = 0.5285,$$

$$\sqrt{a^2 - b^2} = 3.377.$$

§ 6. CIRCULAR, OR TRIGONOMETRIC, FUNCTIONS: NATURAL VALUES.

a. Three tables of the natural values of the trigonometric functions are given on pp. 22-27. Each table is broken up into six divisions, and occupies two pages. The argument is the angle, which is tabulated at intervals of 10' from 0° to 90°. Angles in the *first half of the quadrant* will be found in the *left-hand column* of the several divisions of the table, and for those *angles the names of the functions are to be taken from the top of the page*; *angles in the second half of the quadrant are to be found in the right-hand*

Logarithms of Circular Functions.

column of the table, and for those angles the names of the functions are to be taken from the *bottom* of the page. The angles standing at the right and left in the same line are *complements* of each other; and the names of the *functions* at the top and bottom of the same column are *complementary*. The value of any of the functions for a non-tabulated angle, or the value of the angle for a non-tabulated value of one of the functions, can be found by the method of interpolation explained in § 2. The precepts of § 2, *d*, *e*, should be observed in computing the corrections. The tabulated values of the functions are generally given to four significant figures; but, in the tables of tangents and secants, they are sometimes given to a less number of figures (to avoid errors in interpolation), and are sometimes omitted altogether. In these cases the functions can be best found by finding their logarithms by the table of *Logarithms of Circular Functions* (see § 7), and then the numbers corresponding by the table of *Logarithms*.

b. To find any function of an angle *greater than* 90° , we must subtract from the given angle the greatest multiple of 90° which it contains; if an *even* multiple has been subtracted, we look out the required function of the remainder; if an *odd* multiple, the complementary function; and we then fix the *sign* of the function by considering the *quadrant* in which the given angle lies. For a *negative* angle, we find the required function of the corresponding positive angle, and then fix its sign by considering the quadrant of the angle.

c. Examples of the use of these tables:—

$$\begin{array}{llll} \sin 77^\circ 37' & = & 0.9767, & \tan 53^\circ 04' = 1.330, \quad \sec 68^\circ 45' = 2.759, \\ \cos 16^\circ 19' & = & 0.9597, & \cot 3^\circ 18' = 17.4, \quad \csc 55^\circ 13' = 1.217; \end{array}$$

$$\begin{array}{llll} \sin 257^\circ 37' & = & -0.9767, & \tan 93^\circ 18' = -17.4, \quad \sec 325^\circ 13' = 1.217, \\ \cos 163^\circ 41' & = & -0.9597, & \cot 323^\circ 04' = -1.330, \quad \csc 158^\circ 45' = 2.759; \end{array}$$

$$\begin{array}{llll} \sin (-257^\circ 37') & = & 0.9767, & \tan (-93^\circ 18') = 17.4, \quad \sec (-325^\circ 13') = 1.217, \\ \cos (-163^\circ 41') & = & -0.9597, & \cot (-323^\circ 04') = 1.330, \quad \csc (-158^\circ 45') = -2.759; \end{array}$$

$$\sin^{-1} 0.2000 = 11^\circ 32' \text{ or } = 168^\circ 28' \text{ or } = 371^\circ 32', \text{ etc.},$$

$$\cos^{-1} (-0.3542) = 110^\circ 45' \text{ or } = 249^\circ 15' \text{ or } = 830^\circ 45', \text{ etc.},$$

$$\tan^{-1} (-4.570) = 102^\circ 21' \text{ or } = 282^\circ 21' \text{ or } = -77^\circ 39', \text{ etc.},$$

$$\cot^{-1} 0.3163 = 72^\circ 27' \text{ or } = 252^\circ 27' \text{ or } = -107^\circ 33', \text{ etc.},$$

$$\sec^{-1} 5.000 = 78^\circ 28' \text{ or } = -78^\circ 28' \text{ or } = \pm 281^\circ 32', \text{ etc.},$$

$$\csc^{-1} (-3.529) = -16^\circ 28' \text{ or } = 196^\circ 28' \text{ or } = -163^\circ 32', \text{ etc.}$$

§ 7. LOGARITHMS OF CIRCULAR FUNCTIONS.

To find the logarithm of any circular function of a given angle.

a. If the angle is *less than* 6° , the part of the table which occupies the upper half of p. 10 may be used. (See also *g.*) The *left-hand division* of this part of the table gives the values of a logarithm *S* (the characteristic and the first two figures of the mantissa being printed at the head of the column), with the angular limits between which each value may be used. Thus, for all positive angles less than $1^\circ 51'.479$, $S = 6.4437$; for all angles between $1^\circ 51'.479$ and $2^\circ 49'.567$, $S = 6.4436$; etc. The next following

Explanation of the Tables.

division gives, in like manner, the values of a logarithm T. We must find the logarithm of the angle, reduced to minutes and decimals of a minute, and must then apply the formulas :—

$$\log \sin \phi = \log (\phi \text{ in minutes}) + S - 10,$$

$$\log \tan \phi = \log (\phi \text{ in minutes}) + T - 10.$$

The two right-hand divisions of this part of the table give the values of the log sec, with the angular limits for each value. The logarithms of the co-sine, cotangent, and cosecant are the *arithmetical complements* (— 10) of the logarithms of the secant, tangent, and sine, respectively. Example :—

$$\begin{aligned} 1 \sin 3^\circ 15'.23 &= 8.7541, & 1 \tan 3^\circ 15'.23 &= 8.7548, & 1 \sec 3^\circ 15'.23 &= 0.0007, \\ 1 \csc 3^\circ 15'.23 &= 1.2459, & 1 \cot 3^\circ 15'.23 &= 1.2452, & 1 \cos 3^\circ 15'.23 &= 9.9993; \end{aligned}$$

the negative characteristics being here, as in the following examples, made positive by the addition of 10.

b. If the angle is *acute and greater than 84°*, we must take its complement, and then seek the function complementary to that required, for the angle thus obtained, by the method just expounded. Example :—

$$\begin{aligned} 1 \sin 86^\circ 44'.77 &= 9.9993, & 1 \tan 86^\circ 44'.77 &= 1.2452, & 1 \sec 86^\circ 44'.77 &= 1.2459, \\ 1 \csc 86^\circ 44'.77 &= 0.0007, & 1 \cot 86^\circ 44'.77 &= 8.7548, & 1 \cos 86^\circ 44'.77 &= 8.7541. \end{aligned}$$

c. If the angle is contained *between 6° and 84°*, we use the main part of the table, occupying the lower half of p. 10 and pp. 11–15. The angle is tabulated at intervals of 10', from 6° to 45° in the left-hand column of the table, and from 45° to 84° in the right-hand column. The names of the functions are to be taken from the *tops* of the columns, when the angle is on the *left*; and from the *bottoms* of the columns, when the angle is on the *right*. The angles on the right and left of any line and the names at the top and bottom of any column have the same relation to each other as in the tables of *Natural Values* (§ 6). The true characteristic in the first, third, and sixth columns is —1, but is printed 9. The six columns are arranged in pairs. The two functions in each pair of columns are *reciprocal* to each other; and the logarithms are therefore *complementary*, and their differences are equal in value, with opposite signs. Down the middle of each double column are printed, in small type, the *rates of difference* of the logarithms in that double column. Each value of this rate may be used in interpolation, instead of Δu , through half the interval before and after the line on which it stands, as stated in § 2, c. Thus, in finding the logarithms of the circular functions of any angle between 25° 25' and 25° 35' we work from the values corresponding to 25° 30', the nearest tabulated angle; and compute the corrections by taking proportional parts of 26, 33, and 6, for the three pairs of functions. In applying the corrections, we must carefully observe, for each function, whether the function is increasing or decreasing.

For example, let the logarithms of the circular functions of 25° 27'.4 = 25° 30' — 02'.6 be required. We find

$$\begin{array}{lll} 1 \sin 25^\circ 30' &= 9.6340 & 1 \tan 25^\circ 30' &= 9.6785 & 1 \sec 25^\circ 30' &= 0.0446 \\ 0.26 \times 26 &= 7 & 0.26 \times 33 &= 9 & 0.26 \times 6 &= 2 \\ 1 \sin 25^\circ 27'.4 &= 9.6333, & 1 \tan 25^\circ 27'.4 &= 9.6776, & 1 \sec 25^\circ 27'.4 &= 0.0443, \end{array}$$

Logarithms of Circular Functions.

$l \csc 25^\circ 30' = 0.3660$	$l \csc 25^\circ 30' = 0.3660$	$l \csc 25^\circ 30' = 0.3660$
7	9	2
$l \csc 25^\circ 27'.4 = 0.3667,$	$l \csc 25^\circ 27'.4 = 0.3667,$	$l \csc 25^\circ 27'.4 = 0.3667,$

In like manner, we have

$l \sin 74^\circ 46' = 9.9845,$	$l \tan 74^\circ 46' = 0.5649,$	$l \sec 74^\circ 46' = 0.5804,$
$l \csc 74^\circ 46' = 0.0155,$	$l \csc 74^\circ 46' = 0.0155,$	$l \csc 74^\circ 46' = 0.0155,$

d. If the angle is greater than 90° , or negative, we must use the method explained in § 6, *b*, for the tables of *Natural Values* of the circular functions. When the natural value of a circular function is negative, this should be indicated by writing the letter *n* after its logarithm. Examples :

$l \sin 105^\circ 14' = 9.9845,$	$l \tan 105^\circ 14' = 0.5649 n,$
$l \sec 105^\circ 14' = 0.5804 n,$	
$l \csc 164^\circ 46' = 0.5804,$	$l \csc 164^\circ 46' = 0.5804 n,$
$l \cos 164^\circ 46' = 9.9845 n,$	
$l \sin (-74^\circ 46') = 9.9845 n,$	$l \tan (-105^\circ 14') = 0.5649,$
$l \cos (-394^\circ 46') = 9.4196.$	

Given the logarithm of any circular function, to find the value of the corresponding angle.

e. If the given logarithm lies *without the limits of the main part of the table*, the upper part of p. 10 may be used. If the given logarithm is a *log sin* less than 9.0192, or a *log tan* less than 9.0216, subtract from it the proper value of *S* or *T* (or add the arithmetical complement), and the remainder is the log of the required angle in minutes. The limiting values of the log sin and log tan for each value of *S* and *T* are given in the table. If the given log is a *log csc* greater than 0.9808, or a *log ctn* greater than 0.9784, its arithmetical complement will be a log sin less than 9.0192, or a log tan less than 9.0216. If the given log is a *log sec* less than 0.0024, the limits between which the required angle lies are given by the table; the angle may have any value between these limits, and is not therefore very closely determined. If the given log is a *log cos* greater than 9.9976, its arithmetical complement is a log sec less than 0.0024.

If the given log is a *log sin*, *log tan*, or *log sec* greater than 9.9976, 0.9784, or 0.9808 (respectively), or a *log csc*, *log ctn*, or *log cos* less than 0.0024, 9.0216, or 9.0192 (respectively), we must change the name of the function to the complementary name (sin to cos, etc.), find the corresponding angle as above, and take the complement of the angle thus found. Examples :—

$(\log \sin)^{-1} 8.9542 = 5^\circ 09'.8,$	$(\log \csc)^{-1} 2.0531 = 0^\circ 30'.42,$
$(\log \tan)^{-1} 9.0024 = 84^\circ 15'.5,$	$(\log \sec)^{-1} 9.9983 = 84^\circ 58' \pm 44'.$

f. If the given logarithm is contained *within the limits of the main part of the table*, the required angle is found by ordinary interpolation; and we may use the printed rate of difference as the value of Δu , working in each case from the nearest tabulated value. The angle should be found to the nearest minute, or, when the difference exceeds 100, to the nearest tenth of a

Explanation of the Tables.

minute. But in the right-hand pair of columns, the last figure of the angle thus found will generally be uncertain. Examples:—

Let it be required to find $(\log \sec)^{-1} 0.0343$; i. e. the angle of which the log sec is 0.0643. The nearest tabulated log sec is 0.0647. We have, then,

$$(\log \sec)^{-1} 0.0647 = 30^\circ 30', u_2 - u = 4, \Delta u = 7, \frac{4}{7} = 0.6,$$

$$\therefore (\log \sec)^{-1} 0.0643 = 30^\circ 30' - 03' = 30^\circ 24'.$$

In like manner, let the student find

$$(\log \sin)^{-1} 9.5663 = 21^\circ 37',$$

$$(\log \cotn)^{-1} 0.0496 = 41^\circ 44',$$

$$(\log \cos)^{-1} 9.9188 = 33^\circ 58' \text{ or } 57',$$

$$(\log \sec)^{-1} 0.2372 = 53^\circ 39',$$

$$(\log \tan)^{-1} 0.7507 = 79^\circ 56',$$

$$(\log \csc)^{-1} 0.1433 = 45^\circ 58'.$$

The angle may also be found by the next following table.

g. Pp. 8 and 9 may also be used for angles less than 6° or greater than 84° .

E.g. $1 \sin 4^\circ 03'.4 = 8.8497$, $1 \tan 4^\circ 03'.4 = 8.8508$, $1 \sec 4^\circ 03'.4 = 0.0011$,

$1 \csc 4^\circ 03'.4 = 1.1503$, $1 \cotn 4^\circ 03'.4 = 1.1492$, $1 \cos 4^\circ 03'.4 = 9.9989$.

§ 8. INVERSE CIRCULAR FUNCTIONS.

a. The table having this heading (pp. 16–18) is a table for finding the angle which corresponds to the given logarithm of a circular function. The logarithm (increased by 10) is the *argument* of the table, and is to be regarded as given to *four places* of decimals. It is tabulated at intervals of 0.0100 from 9.0000 to 0.0000 through the first page of the table, then at intervals of 0.0010, and in the last two divisions at intervals of 0.0001. The characteristic of the argument is printed at the head of the column. The figures supposed to follow the printed figures in the values of the argument are zeros. Thus, the first value is 9.0000, the next 9.0100, etc. The angle is given, for convenience of interpolation, in degrees and decimals of a degree. When found, it is easily reduced to degrees and minutes, if that is necessary, and should, in general, be taken *only to the nearest minute*. The angle under the heading $\sin^{-1} u$ is that angle of which the corresponding value of the argument, $\log u$, is the log sin; etc. In interpolating in this table, we may use the *printed rate of difference* instead of Δu , working from the *nearest* tabulated value of the argument, and carefully observing whether the tabulated angle ought to be *increased* or *diminished*. When the *printed rate of difference* is omitted, this is because the interval is too great to admit of accurate interpolation. In this case, we must resort to those later divisions of the table in which the argument is tabulated at smaller intervals. When the last figure of the tabulated angle is printed in small type, this shows that that figure is uncertain, if the logarithm is given to only four places; that is, that there is a possible variation, on each side of the tabulated angle, as great as *half a unit* in the place of the figure so printed. For example, if $\log u = 9.9000$, we find the last figures of $\sin^{-1} u$ and $\cos^{-1} u$ to be printed in small type. Now, seven-place tables show that $(\log \sin)^{-1} 9.8999500 = 52^\circ.581$, while $(\log \sin)^{-1} 9.9000500 = 52^\circ.600$. But 9.9000 may represent any logarithm between these; and hence the corresponding angle, in this case, admits a like variation, while $\cos^{-1} u$ may have any value between $37^\circ.419$ and $37^\circ.400$.

Hyperbolic Functions.

Neither of these difficulties presents itself in finding an angle from its $\log \tan$ or $\log \csc$. If $\log u = 9.9000$, $\tan^{-1} u$ can only vary from $38^\circ.458$ to $38^\circ.464$.

The angle found by interpolation should be carried out *only to the nearest hundredth of a degree, in any case*. The last column of the table shows that the angle is not always determined even to the nearest *tenth*.

b. If the characteristic of the given logarithm is 0, we must take its arithmetical complement, which will be the logarithm of the reciprocal function of the same angle. The angle can then be found by the table.

c. If the given logarithm is less than 9.0000, or greater than 0.0000, the tables in the upper part of p. 10 may be used, as explained in § 7, e; or pp. 8, 9.

d. Let us find by this table the angles sought above, in § 7, f. We have, in the case of the first example,

$$(\log \sec)^{-1} 0.0643 = (\log \cos)^{-1} 9.9357.$$

Then the table gives

$$(\log \cos)^{-1} 9.9360 = 30^\circ.35, \quad \Delta u = 23,$$

$$0.3 \times 0.23 = .07$$

$$\therefore (\log \cos)^{-1} 9.9357 = 30^\circ.42 = 30^\circ 25'.$$

In fact, the limits of the angle are $30^\circ 24'.2$ and $30^\circ 25'.6$, the mean value being $30^\circ 24'.9$. In this case, the present table gives a better value than the other; but both values are admissible.

In like manner, we have

$$(\log \sin)^{-1} 9.5663 = 21^\circ.81 - 20 = 21^\circ.61 = 21^\circ 37',$$

$$(\log \cos)^{-1} 9.9188 = 33^\circ.92 + .04 = 33^\circ.96 = 33^\circ 58',$$

$$(\log \tan)^{-1} 0.7507 = 79^\circ.92 + .02 = 79^\circ.94 = 79^\circ 56',$$

$$(\log \csc)^{-1} 0.0496 = 41^\circ.71 + .03 = 41^\circ.74 = 41^\circ 44',$$

$$(\log \sec)^{-1} 0.2272 = 53^\circ.64 + .02 = 53^\circ.66 = 53^\circ 40',$$

$$(\log \csc)^{-1} 0.1433 = 46^\circ.01 - .04 = 45^\circ.97 = 45^\circ 58'.$$

§ 9. HYPERBOLIC FUNCTIONS.

a. The **hyperbolic functions** are certain functions which bear relations to the equilateral hyperbola similar to those borne by the circular functions to the circle; and they may often be usefully employed both in computation and in analysis. They are named the hyperbolic sine, cosine, tangent, cotangent, secant, and cosecant; and are variously denoted by different writers. They are here represented by the symbols: Sh, Ch, Th, Cth, Sch, Csch. They may be defined by the following formulas, in which

\mathcal{G} = the exponential base

$$= 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \dots$$

$$= 2.7182818285 \dots :-$$

$$\text{Sh } x = \frac{1}{2} (\mathcal{G}^x - \mathcal{G}^{-x}), \quad \text{Ch } x = \frac{1}{2} (\mathcal{G}^x + \mathcal{G}^{-x}), \quad \text{Th } x = \frac{\text{Sh } x}{\text{Ch } x},$$

$$\text{Cth } x = \frac{1}{\text{Th } x}, \quad \text{Sch } x = \frac{1}{\text{Ch } x}, \quad \text{Csch } x = \frac{1}{\text{Sh } x}.$$

Explanation of the Tables.

They bear to the circular functions the relations expressed by the following formulas, in which $i = \sqrt{-1}$:—

$$\begin{array}{ll}
 \text{Sh } x = \frac{\sin xi}{i}, & \sin x = \frac{\text{Sh } xi}{i}, \\
 \text{Ch } x = \cos xi, & \cos x = \text{Ch } xi, \\
 \text{Th } x = \frac{\tan xi}{i}, & \tan x = \frac{\text{Th } xi}{i}, \\
 \text{Cth } x = i \text{ctn } xi, & \text{ctn } x = i \text{Cth } xi, \\
 \text{Sch } x = \sec xi, & \sec x = \text{Sch } xi, \\
 \text{Csch } x = i \csc xi, & \csc x = i \text{Csch } xi.
 \end{array}$$

Again, if ϕ is so taken that

$$x = \text{nat log tan } (45^\circ + \frac{1}{2} \phi),$$

$$\begin{array}{lll}
 \text{Sh } x = \tan \phi, & \text{Ch } x = \sec \phi, & \text{Csch } x = \text{ctn } \phi, \\
 \text{Th } x = \sin \phi, & \text{Sch } x = \cos \phi, & \text{Cth } x = \csc \phi.
 \end{array}$$

The value of ϕ determined by this formula has been called by some writers the **Gudermannian** of x , and denoted by the symbol: $gd\ x$.

b. From $x = 0.00$ to $x = 1.00$, the function tabulated is $gd\ x$ in degrees, at intervals of 0.01 in the value of x . The hyperbolic functions of x are then readily found, by the aid of the formulas last given, from the tables of circular functions. Beginning with $x = 1.00$, $\log \text{Sh } x$, $\log \text{Ch } x$, and $\log \text{Th } x$ are tabulated, at intervals of 0.01 in the value of x , up to $x = 3.00$, the characteristic of each logarithm being placed at the head of its column; then at intervals of 0.1 up to $x = 6.0$; and lastly at intervals of 1 up to $x = 10.0$. The printed differences are to be used, as in other tables, each through *half the interval* before and after the line on which it stands.

c. If $x > 10$, $\log \text{Th } x = 0.0000$, while $\log \text{Sh } x$ and $\log \text{Ch } x$ may be found by the formula and table given at the lower right-hand corner of p. 21. The quantity μ is the *modulus* of the denary system of logarithms; that is, it is the denary logarithm of the exponential base. The values of $n\mu$ being given for all integral values of n from 1 to 10, any product $x\mu$ is readily found, by adding together the products of μ by the successive figures of x . Only four decimal-figures should be retained in the result.

d. The functions $\log \text{Cth } x$, $\log \text{Sch } x$, and $\log \text{Csch } x$ are the *arithmetical complements* of $\log \text{Th } x$, $\log \text{Ch } x$, and $\log \text{Sh } x$, respectively.

e. The table may be used both directly and inversely. Examples:—

$$\begin{array}{ll}
 \log \text{Sh } 0.5378 & = \log \tan 29^\circ.427 = 9.7513, \\
 \log \text{Ch } 1.5280 & = 0.3825, \\
 \log \text{Cth } 1.8240 & = 0.0226, \\
 \log \text{Sh } 12.5913 & = \log \text{Ch } 12.5913 = 5.1673, \\
 (\log \text{Ch})^{-1} 1.6000 & = 4.377, \\
 (\log \text{Sh})^{-1} 5.0000 & = 12.206, \\
 (\log \text{Th})^{-1} 9.9012 & = 1.089, \\
 (\log \text{Sch})^{-1} 9.5873 & = 1.604, \\
 (\log \text{Csch})^{-1} 0.3924 & = gd^{-1} (\log \tan)^{-1} 9.8078 \\
 & = gd^{-1} 22^\circ.06 = 0.396.
 \end{array}$$

Natural Logarithms

§ 10. NATURAL LOGARITHMS.

a. The natural system of logarithms is that which is founded on the exponential base (see § 9). This number is defined as the limiting value to which the expression

$$(1 + \epsilon)^{\frac{1}{\epsilon}} = \sqrt[\epsilon]{1 + \epsilon}$$

approaches, as ϵ approaches 0. It is most frequently denoted by the letter e ; but, as being one of the few peculiar constants of analysis, it is here represented by the symbol \mathcal{G} , which may be read "base."

The following formulas are proved in treatises on the Differential Calculus:—

$$\mathcal{G} = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \dots,$$

$$\mathcal{G}^x = 1 + \frac{x}{1} + \frac{x^2}{1.2} + \frac{x^3}{1.2.3} + \frac{x^4}{1.2.3.4} + \dots,$$

$$\text{nat log } (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots;$$

the second formula being applicable to *all values* of x , but the last only when x is *numerically less than 1*. If x is *very small*, then *approximately*

$$\mathcal{G}^x = 1 + x, \quad \text{nat log } (1 + x) = x, \quad \text{nat log } (1 - x) = -x.$$

We also have, in the natural system,

$$\log (a + h) = \log a + \log \left(1 + \frac{h}{a} \right) = \log a + \frac{h}{a} - \frac{h^2}{2a^2} + \frac{h^3}{3a^3} - \dots,$$

provided h is numerically less than a .

The rate of difference of \mathcal{G}^x , for $\Delta x = 1$, is always \mathcal{G}^x , and that of $\text{nat log } x$ is $\frac{1}{x}$.

b. The numerical value of \mathcal{G} or of any power of \mathcal{G} can be computed, to any assigned number of decimal-places, by using a sufficient number of terms of the first two series given above. Thus, to find \mathcal{G} to four decimal-places, we proceed as follows, observing that, if any term be divided by its number in the series, the next following term is obtained:—

$$\begin{array}{r} 1) 1.00000 \\ 2) 1.00000 \\ 3) 0.50000 \\ 4) 0.16667 \\ 5) 0.04167 \\ 6) 0.00833 \\ 7) 0.00139 \\ 8) 0.00020 \\ \quad 0.00002 \\ \hline \mathcal{G} = 2.7183 \dots \end{array}$$

Explanation of the Tables.

c. The **modulus** of any system of logarithms is the logarithm of \odot in that system. If m is the modulus of a system of which a is the base, then

$$a^m = \odot, \quad \odot^{m^{-1}} = a.$$

The modulus of the *natural system itself* is 1. The values of the modulus of the *denary system* and of the reciprocal of that modulus are

$$\begin{aligned} \mu &= \text{den log } \odot = 0.4342944819 \dots, \\ \mu^{-1} &= \text{nat log } 10 = 2.3025850930 \dots \end{aligned}$$

By the rule for converting logarithms from one system to another, the logarithm of a number in any system may be found by *multiplying the modulus of that system into the natural logarithm of the same number*. Thus,

$$\begin{aligned} \text{den log } x &= \mu \text{ nat log } x, \\ \text{nat log } x &= \mu^{-1} \text{ den log } x. \end{aligned}$$

By the aid of these formulas, the table at the bottom of p. 21 may be used to find the natural logarithm of any number, or the denary logarithm of any power of the exponential base, or to find a number from its natural logarithm. For example:—

$$\begin{aligned} \text{nat log } 72.5 &= 1.8603 \times \mu^{-1} = 4.2835, \\ \text{nat log } 1.0074 &= 0.0032 \times \mu^{-1} = 0.0074, \\ \text{den log } \odot^{\frac{1}{4}} &= \frac{1}{4}\mu = 0.0620, \\ (\text{nat log})^{-1} 10.2108 &= (\text{den log})^{-1} (10.2108 \times \mu) \\ &= (\text{den log})^{-1} 4.4345 = 27194. \end{aligned}$$

d. The natural system is so called, because, in the higher mathematics, it is convenient to regard all other systems as founded upon this. It is named by some writers **hyperbolic**, and by others **Neperian**. But in fact, it is not the system of Napier; nor has it any other relation to the hyperbola than that which belongs to logarithms in general.

e. We may make the following statement of the relation of logarithms and of the hyperbolic functions to the hyperbola, using the notation of Analytic Geometry:—

Let $xy = 1$ be the equation of an hyperbola referred to its asymptotes. It can be proved by the Integral Calculus that the *area*, contained between the curve and the axis of x , and between two ordinates of which one is drawn to the vertex of the curve, is measured by $\log x$ in the system of which the modulus is $\sin \omega$. Thus, *the logarithms belonging to any system may be represented by the areas of an appropriate hyperbola*. The natural system corresponds to the equilateral hyperbola, for which $\sin \omega = 1$.

Again, if u denotes twice the area of the *sector* of the hyperbola $x^2 - y^2 = 1$, contained between the axis of x and a radius vector from the centre, then

$$x = \text{Ch } u, \quad y = \text{Sh } u;$$

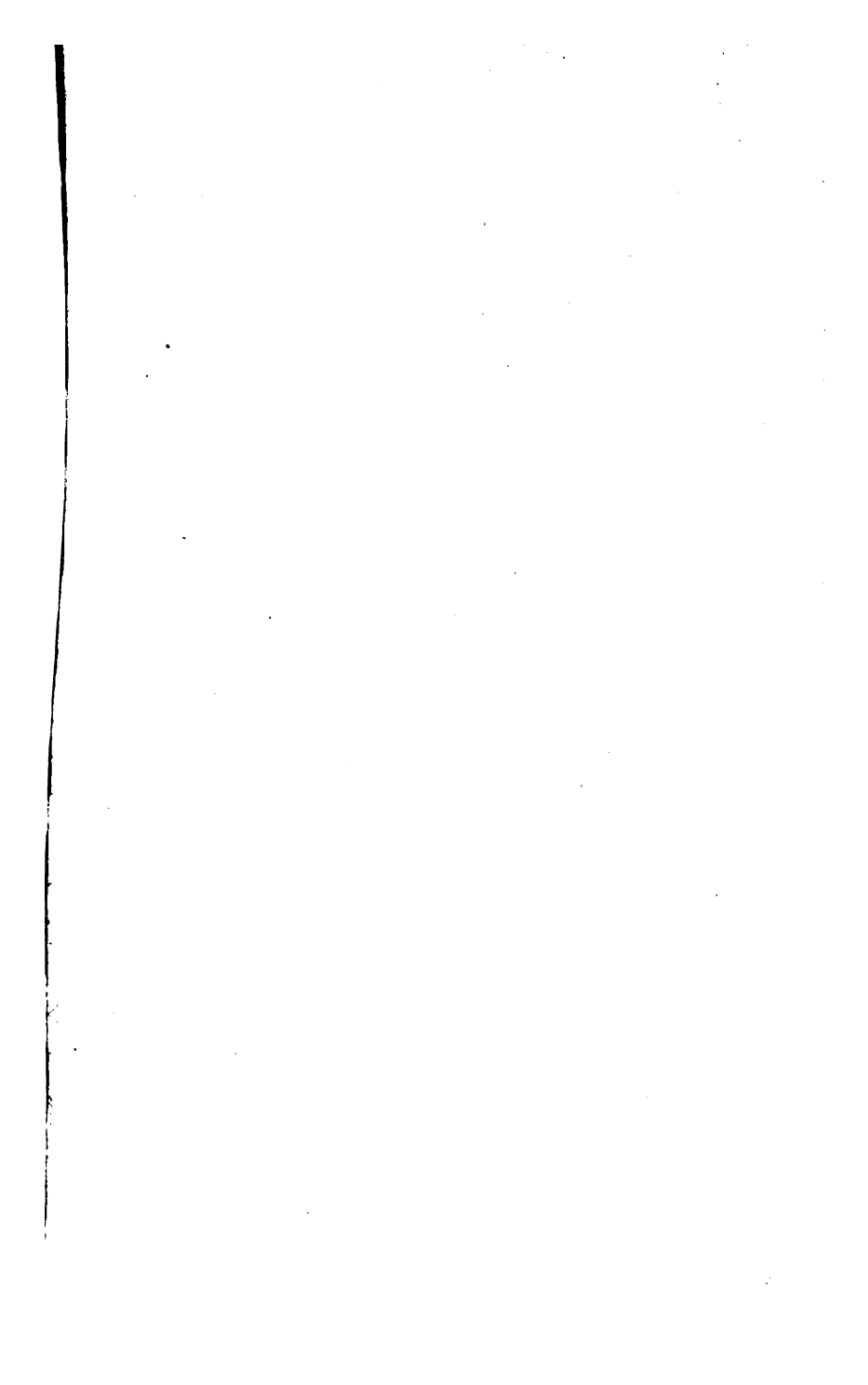
just as, in the circle $x^2 + y^2 = 1$, with a similar meaning of u ,

$$x = \cos u, \quad y = \sin u.$$

Proportional Parts.

10					15					20				
0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	1	
1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	2	
2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0	3	
3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0	6.4	6.8	7.2	7.6	8.0	4	
4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.6	8.0	8.5	9.0	9.5	10.0	5	
4.8	5.4	6.0	6.6	7.2	7.8	8.4	9.0	9.6	10.2	10.8	11.4	12.0	6	
5.6	6.3	7.0	7.7	8.4	9.1	9.8	10.5	11.2	11.9	12.6	13.3	14.0	7	
6.4	7.2	8.0	8.8	9.6	10.4	11.2	12.0	12.8	13.6	14.4	15.2	16.0	8	
7.2	8.1	9.0	9.9	10.8	11.7	12.6	13.5	14.4	15.3	16.2	17.1	18.0	9	
30					35					40				
2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	1	
5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0	7.2	7.4	7.6	7.8	8.0	2	
8.4	8.7	9.0	9.3	9.6	9.9	10.2	10.5	10.8	11.1	11.4	11.7	12.0	3	
11.2	11.6	12.0	12.4	12.8	13.2	13.6	14.0	14.4	14.8	15.2	15.6	16.0	4	
14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0	5	
16.8	17.4	18.0	18.6	19.2	19.8	20.4	21.0	21.6	22.2	22.8	23.4	24.0	6	
19.6	20.3	21.0	21.7	22.4	23.1	23.8	24.5	25.2	25.9	26.6	27.3	28.0	7	
22.4	23.2	24.0	24.8	25.6	26.4	27.2	28.0	28.8	29.6	30.4	31.2	32.0	8	
25.2	26.1	27.0	27.9	28.8	29.7	30.6	31.5	32.4	33.3	34.2	35.1	36.0	9	
50					55					60				
4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	1	
9.6	9.8	10.0	10.2	10.4	10.6	10.8	11.0	11.2	11.4	11.6	11.8	12.0	2	
14.4	14.7	15.0	15.3	15.6	15.9	16.2	16.5	16.8	17.1	17.4	17.7	18.0	3	
19.2	19.6	20.0	20.4	20.8	21.2	21.6	22.0	22.4	22.8	23.2	23.6	24.0	4	
24.0	24.5	25.0	25.5	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	5	
28.8	29.4	30.0	30.6	31.2	31.8	32.4	33.0	33.6	34.2	34.8	35.4	36.0	6	
33.6	34.3	35.0	35.7	36.4	37.1	37.8	38.5	39.2	39.9	40.6	41.3	42.0	7	
38.4	39.2	40.0	40.8	41.6	42.4	43.2	44.0	44.8	45.6	46.4	47.2	48.0	8	
43.2	44.1	45.0	45.9	46.8	47.7	48.6	49.5	50.4	51.3	52.2	53.1	54.0	9	
70					75					80				
6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0	1	
13.6	13.8	14.0	14.2	14.4	14.6	14.8	15.0	15.2	15.4	15.6	15.8	16.0	2	
20.4	20.7	21.0	21.3	21.6	21.9	22.2	22.5	22.8	23.1	23.4	23.7	24.0	3	
27.2	27.6	28.0	28.4	28.8	29.2	29.6	30.0	30.4	30.8	31.2	31.6	32.0	4	
34.0	34.5	35.0	35.5	36.0	36.5	37.0	37.5	38.0	38.5	39.0	39.5	40.0	5	
40.8	41.4	42.0	42.6	43.2	43.8	44.4	45.0	45.6	46.2	46.8	47.4	48.0	6	
47.6	48.3	49.0	49.7	50.4	51.1	51.8	52.5	53.2	53.9	54.6	55.3	56.0	7	
54.4	55.2	56.0	56.8	57.6	58.4	59.2	60.0	60.8	61.6	62.4	63.2	64.0	8	
61.2	62.1	63.0	63.9	64.8	65.7	66.6	67.5	68.4	69.3	70.2	71.1	72.0	9	
90					95					100				
8.8	8.9	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0	1	
17.6	17.8	18.0	18.2	18.4	18.6	18.8	19.0	19.2	19.4	19.6	19.8	20.0	2	
26.4	26.7	27.0	27.3	27.6	27.9	28.2	28.5	28.8	29.1	29.4	29.7	30.0	3	
35.2	35.6	36.0	36.4	36.8	37.2	37.6	38.0	38.4	38.8	39.2	39.6	40.0	4	
44.0	44.5	45.0	45.5	46.0	46.5	47.0	47.5	48.0	48.5	49.0	49.5	50.0	5	
52.8	53.4	54.0	54.6	55.2	55.8	56.4	57.0	57.6	58.2	58.8	59.4	60.0	6	
61.6	62.3	63.0	63.7	64.4	65.1	65.8	66.5	67.2	67.9	68.6	69.3	70.0	7	
70.4	71.2	72.0	72.8	73.6	74.4	75.2	76.0	76.8	77.6	78.4	79.2	80.0	8	
79.2	80.1	81.0	81.9	82.8	83.7	84.6	85.5	86.4	87.3	88.2	89.1	90.0	9	







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